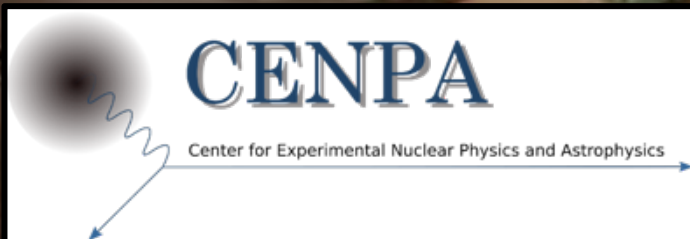


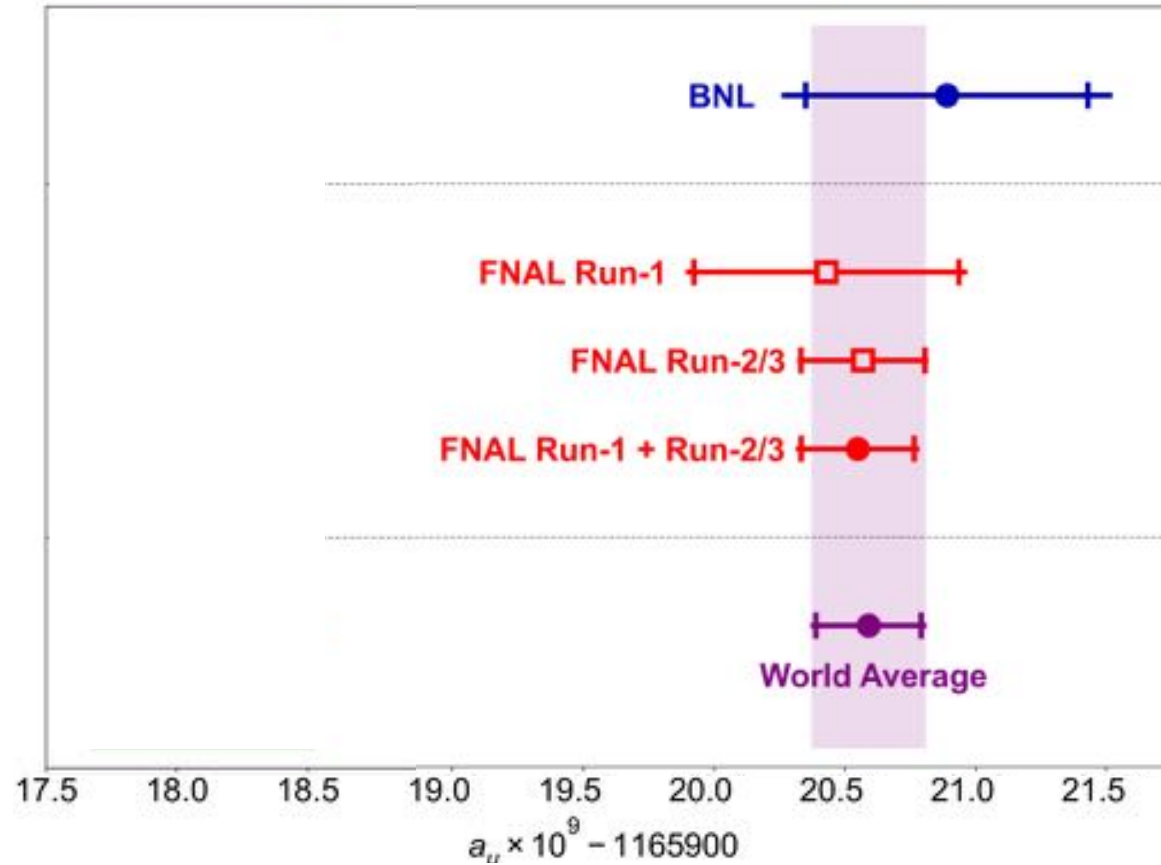
50+ Years of Muon “g-2” experiments



David Hertzog
 University of Washington
 Wojcicki Symposium, Nov. 10, 2023

The Buzz around the Muon $g-2$ Measurement ...

$$a_\mu \equiv \frac{(g - 2)}{2}$$

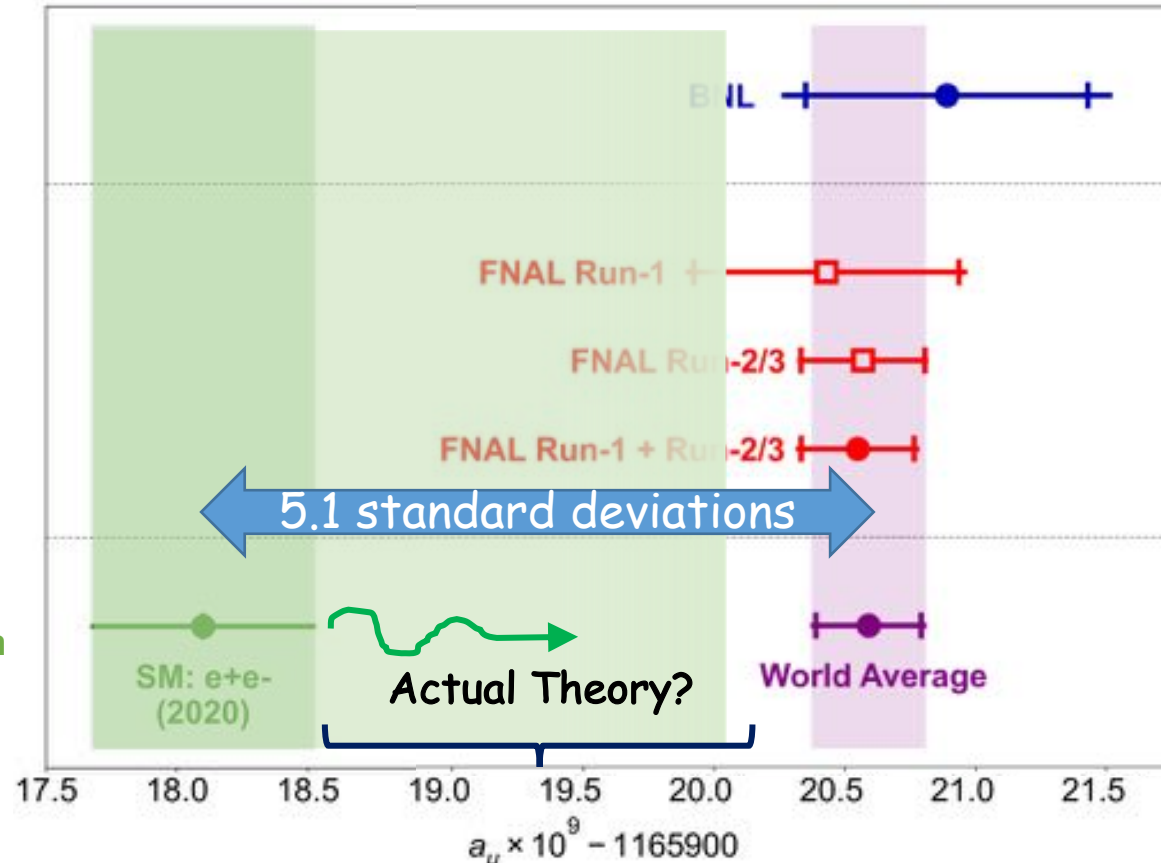


FNAL (2023)
precision ± 200 ppb

The Buzz around the Muon g-2 Measurement ...

$$a_\mu \equiv \frac{(g - 2)}{2}$$

World-wide g-2 Theory Initiative Recommendation (340 ppb)



Looks like a Discovery !!



FNAL (2023) precision ± 200 ppb



Various Wrenches in the Works (to be explained)

A truly radical thought ...

1956

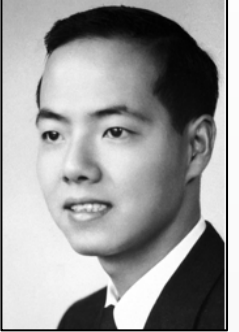
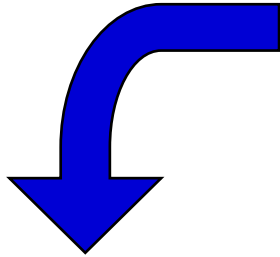
Question of Parity Conservation in Weak Interactions*

T. D. LEE, *Columbia University, New York, New York*

AND

C. N. YANG,† *Brookhaven National Laboratory, Upton, New York*

(Received June 22, 1956)



1957 Nobel prize

Observations of the Failure of Conservation of Parity and Charge Conjugation in Meson Decays: the Magnetic Moment of the Free Muon*

RICHARD L. GARWIN,† LEON M. LEDERMAN, AND MARCEL WEINRICH

Physics Department, Nevis Cyclotron Laboratories, Columbia University, Irvington-on-Hudson, New York, New York

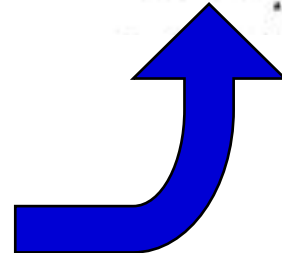
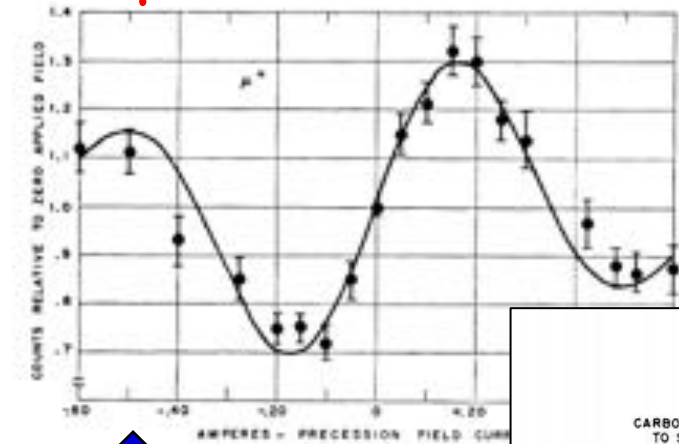
(Received January 15, 1957)

$$\pi^+ \rightarrow \mu^+ + \nu, \quad (1)$$

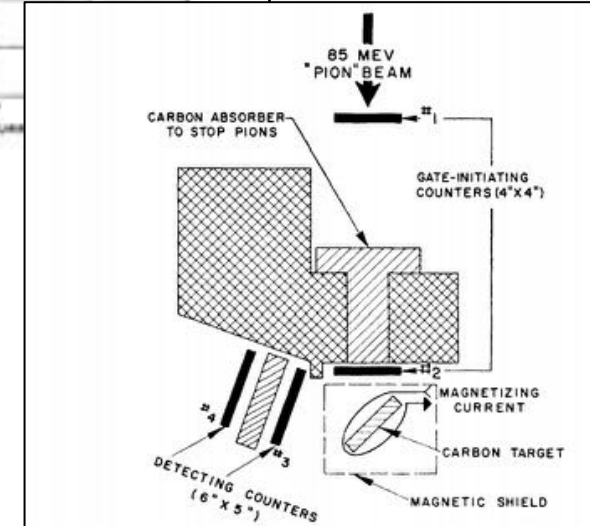
$$\mu^+ \rightarrow e^+ + 2\nu. \quad (2)$$

They have pointed out that parity nonconservation implies a polarization of the spin of the muon emitted from stopped pions in (1) along the direction of motion and that furthermore, the angular distribution of electrons in (2) should serve as an analyzer for the muon polarization. They also point out that the longitudinal polarization of the muons offers a natural way of determining the magnetic moment.⁵ Confirmation of

$$g_\mu = 2.0 \pm 0.1$$



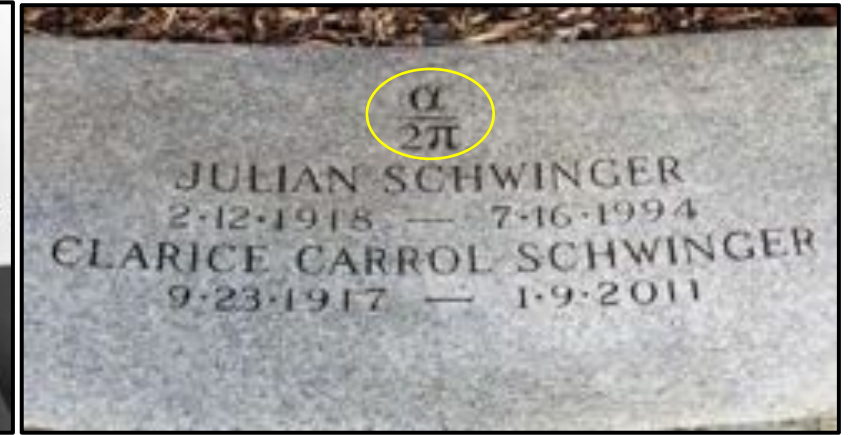
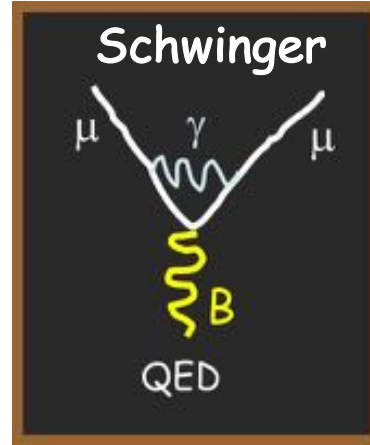
Particles with spin can be Left Handed or Right Handed



In 1947, deviations from $g = 2$ at $\sim 0.1\%$ level observed for the “point-like” electron \rightarrow development of quantum electrodynamics

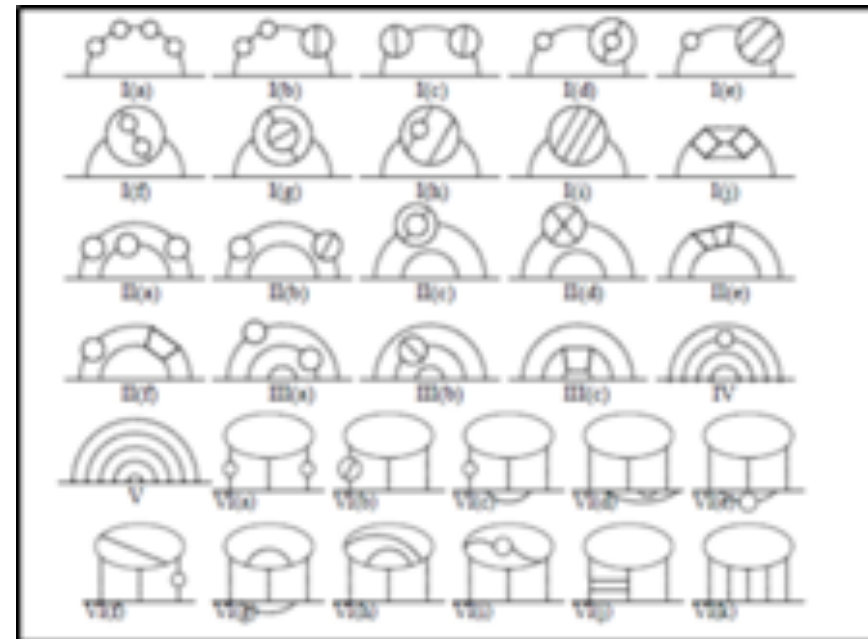
$$a_e = \frac{(g - 2)}{2} \approx \frac{1}{2} \frac{\alpha}{\pi} \approx \frac{1}{800}$$

$$\rightarrow 116\,140\,973.301 \times 10^{-11}$$



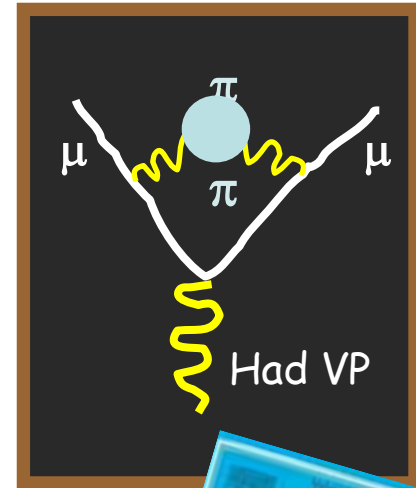
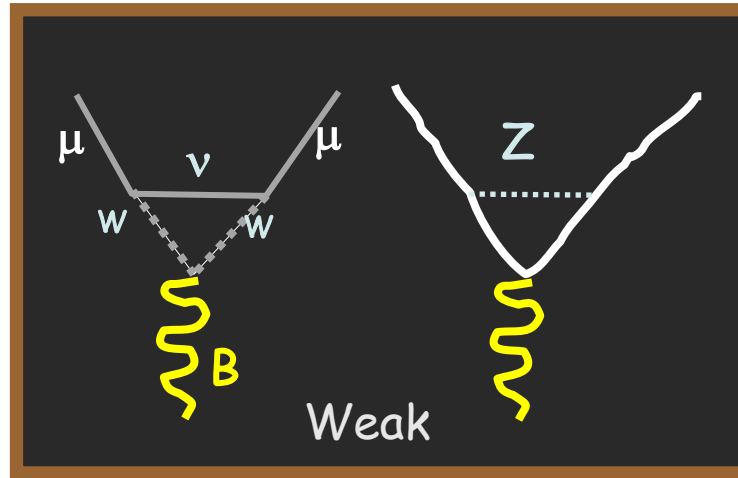
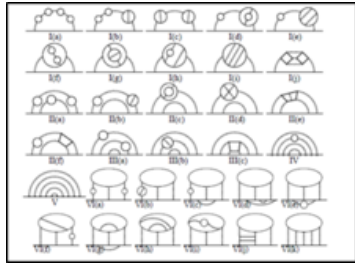
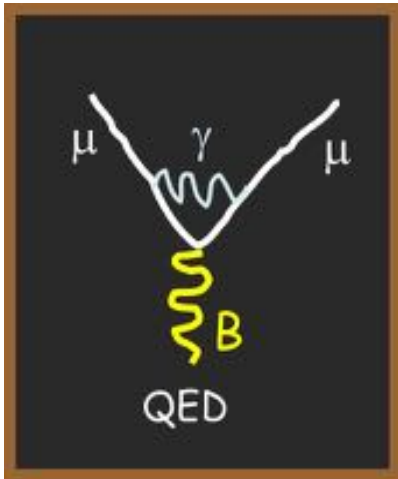
Improved over decades, reaching extraordinary levels of precision

QED 1st Order	116140973.301
QED 2nd Order	413217.621
QED 3rd Order	30141.902
QED 4th Order	380.807
QED 5th Order	4.483
	$\times 10^{-11}$



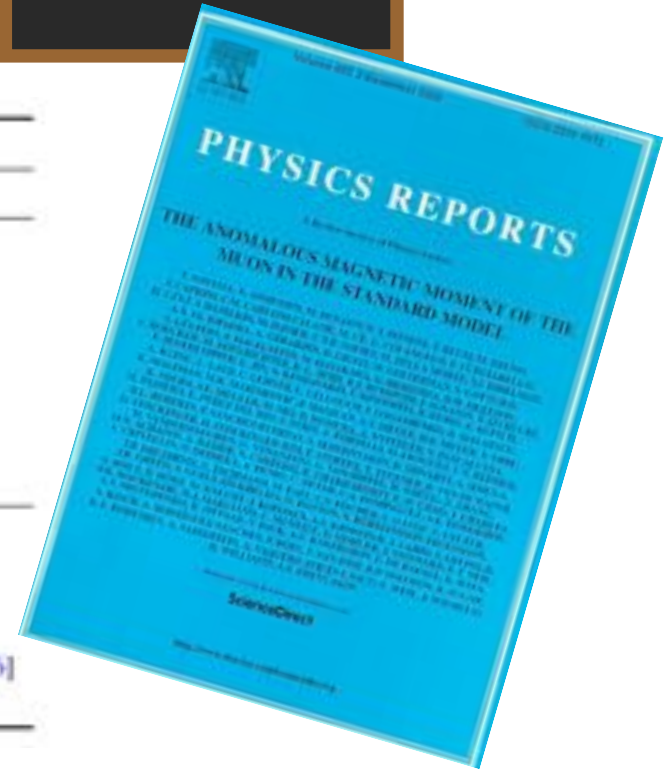
Takeaway: Uncertainty totally negligible

The Muon anomaly is sensitive to all particles that interact with it ...

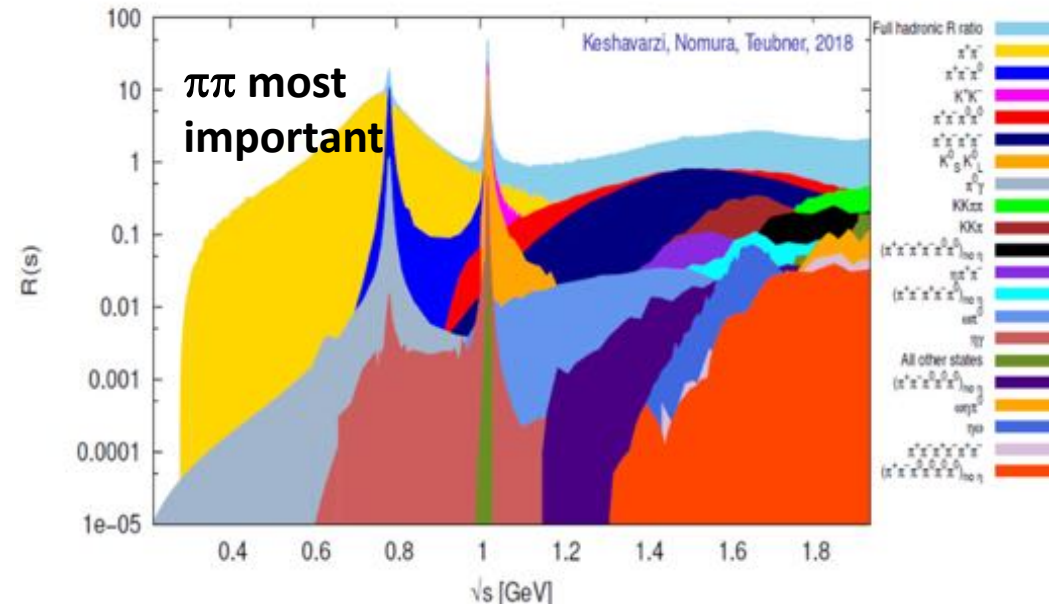
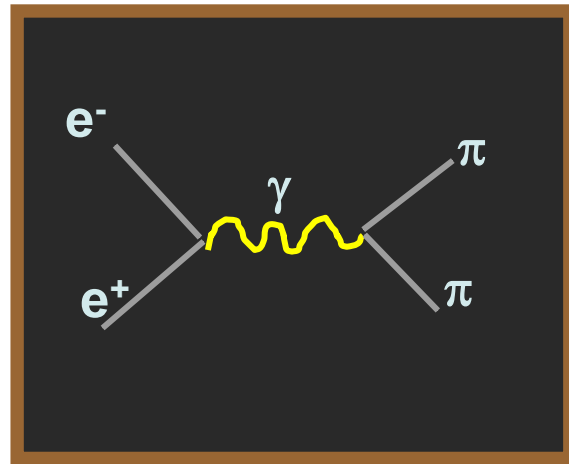
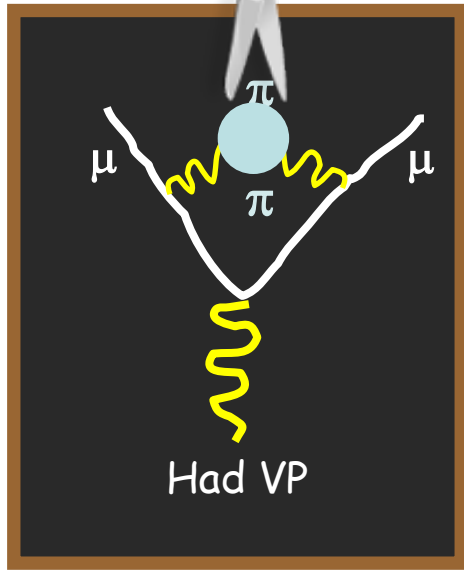


Contribution	Section	Equation	Value $\times 10^{11}$	References
Experiment (E821)		Eq. (8.13)	116 592 089(63)	Ref. [1]
HVP LO (e^+e^-)	Sec. 2.3.7	Eq. (2.33)	6931(40)	Refs. [2–7]
HVP NLO (e^+e^-)	Sec. 2.3.8	Eq. (2.34)	-98.3(7)	Ref. [7]
HVP NNLO (e^+e^-)	Sec. 2.3.8	Eq. (2.35)	12.4(1)	Ref. [8]
HVP LO (lattice, $udsc$)	Sec. 3.5.1	Eq. (3.49)	7116(184)	Refs. [9–17]
HLbL (phenomenology)	Sec. 4.9.4	Eq. (4.92)	92(19)	Refs. [18–30]
HLbL NLO (phenomenology)	Sec. 4.8	Eq. (4.91)	2(1)	Ref. [31]
HLbL (lattice, uds)	Sec. 5.7	Eq. (5.49)	79(35)	Ref. [32]
HLbL (phenomenology + lattice)	Sec. 8	Eq. (8.10)	90(17)	Refs. [18–30, 32]
QED	Sec. 6.5	Eq. (6.30)	116 584 718.931(104)	Refs. [33, 34]
Electroweak	Sec. 7.4	Eq. (7.16)	153.6(1.0)	Refs. [35, 36]
HVP (e^+e^- , LO + NLO + NNLO)	Sec. 8	Eq. (8.5)	6845(40)	Refs. [2–8]
HLbL (phenomenology + lattice + NLO)	Sec. 8	Eq. (8.11)	92(18)	Refs. [18–32]
Total SM Value	Sec. 8	Eq. (8.12)	116 591 810(43)	Refs. [2–8, 18–24, 31–36]

370 ppb



Hadronic Vacuum Polarization is obtained from decades of precise e^+e^- experiments because of simple dispersion relation that connects a_μ to data:



The uncertainty is entirely from experimental errors (statistics dominates)

1. Cut diagram down middle
2. It now looks like $\gamma \rightarrow \pi\pi$

$$a_\mu^{\text{had,LO}} = \frac{\alpha^2(0)}{3\pi^2} \int_{4m_\pi^2}^{\infty} ds \frac{K(s)}{s} R(s) \quad R(s) = \frac{\sigma(e^+e^- \rightarrow \text{hadrons})}{\sigma(e^+e^- \rightarrow \text{muons})}$$

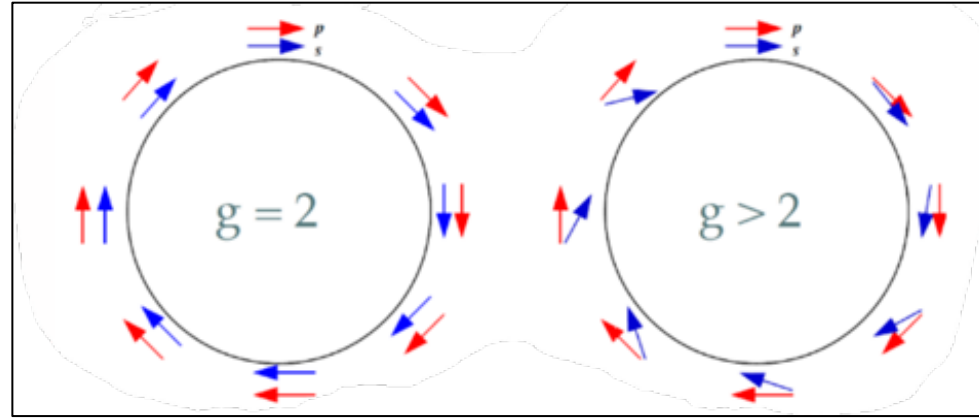
Takeaway: This uncertainty **dominates** SM prediction ... and will become the important end of today's story

The physics of interest is the **small difference** of g from 2 to test the completeness of the SM

$$g_{\mu} = 2.002\,331\,840\,80(11)$$


Design the experiment to **measure this part** to high precision.

The Fundamental Experimental Principle: *In-flight* measurement



$$\omega_C = \frac{eB}{mc\gamma}$$

$$\omega_S = \frac{geB}{2mc} + (1 - \gamma) \frac{eB}{\gamma mc}$$

$$\omega_a = \omega_S - \omega_C = \left(\frac{g-2}{2} \right) \frac{eB}{mc} = a \frac{eB}{mc}$$

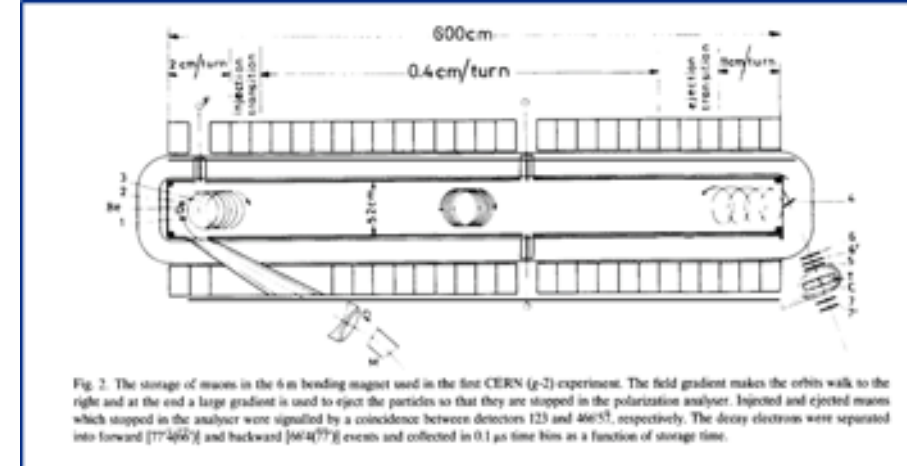
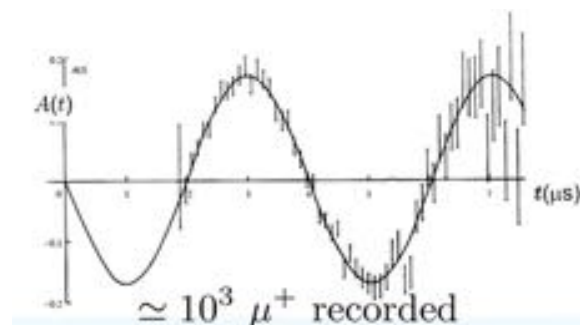
The difference between spin precession frequency and cyclotron frequencies, $\vec{\omega}_a$, does not depend on γ ! Therefore,

$$\vec{\omega}_a = -a_\mu \frac{e}{mc} \vec{B}$$

↑ ↓ ↑

Measure these

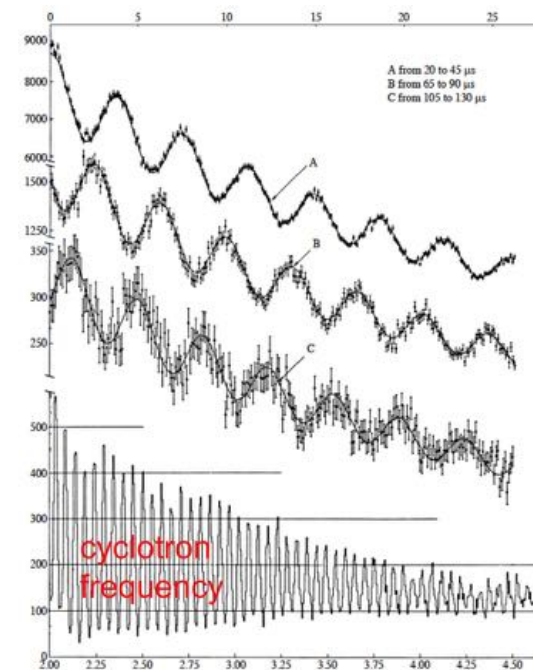
begin to precisely test leading QED predictions



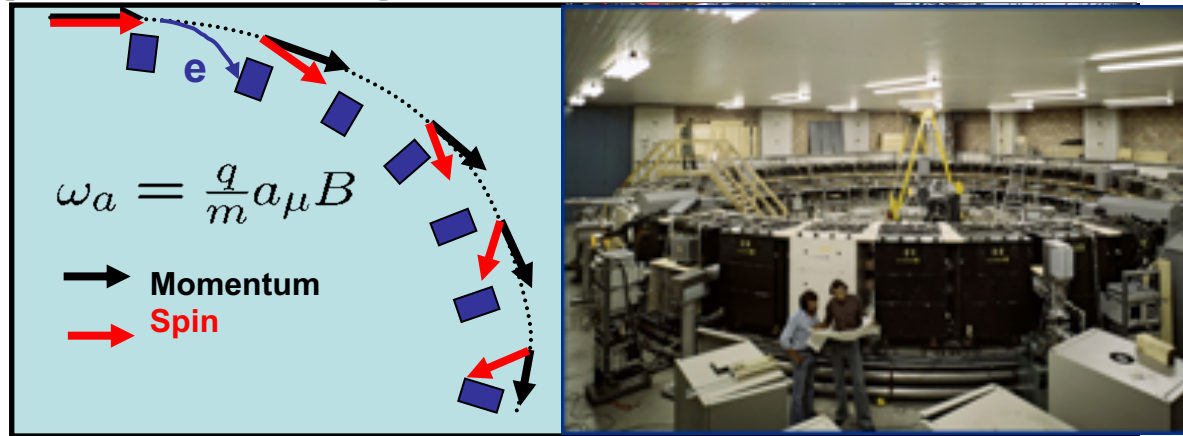
CERN I (1965): precision $\pm 4\,300\,000$ ppb



CERN II (1968): 1st use of a magnetic ring
Precision: 265 000 ppb



The expression is more complicated when you add in E -field (vertical) focusing and out of plane oscillations



THE CERN III EXPERIMENT

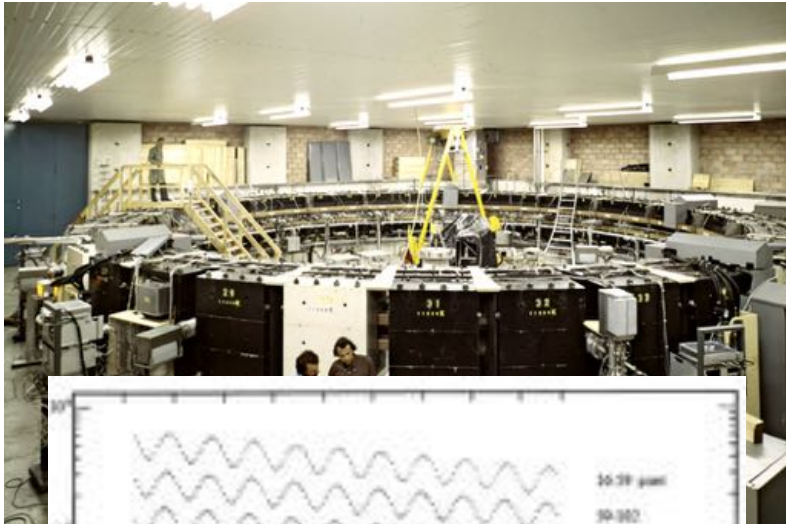
The motion is very nearly planar and the momentum is very nearly the ideal one, but both effects are not perfect and require corrections

$$\vec{\omega}_a = -\frac{q}{m} \left[a_\mu \vec{B} - a_\mu \left(\frac{\gamma}{\gamma + 1} \right) (\vec{\beta} \cdot \vec{B}) \vec{\beta} - \left(a_\mu - \frac{1}{\gamma^2 - 1} \right) \frac{\vec{\beta} \times \vec{\mathcal{E}}}{c} \right]$$

0 if “in plane”

Term cancels at 3.094 GeV/c, the “Magic γ ”

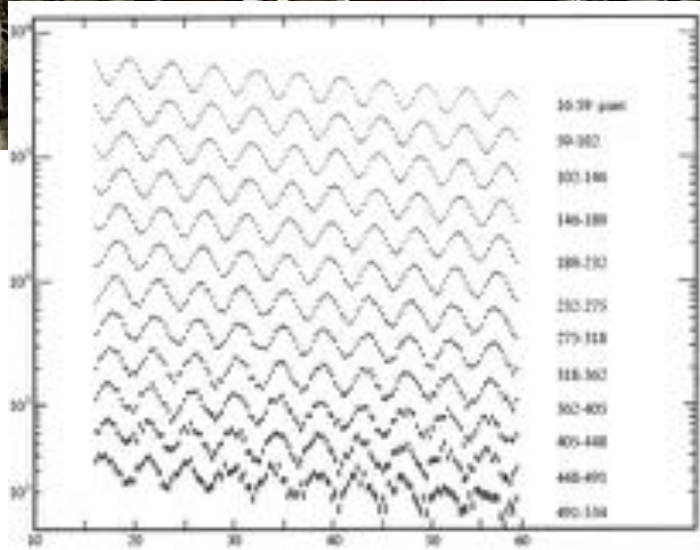
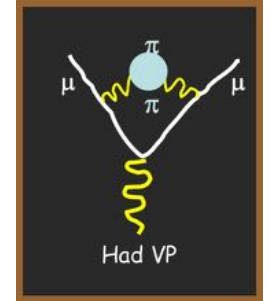
Summary of the CERN III Experiment



Precision: 7000 ppb

The main conclusions:

1. The **QED** calculation of the anomaly is verified up to the **sixth order**, the experimental uncertainty being equivalent to 5% of this term.
2. **The hadronic contribution to the anomaly** is observed and measured to an accuracy of 20%
3. There is no evidence for a special coupling of the muon.



Final Report on the CERN Muon Storage Ring Including the Anomalous Magnetic Moment and the Electric Dipole Moment of the Muon, and a Direct Test of Relativistic Time Dilation; Nucl.Phys.B 150 (1979) 1-75

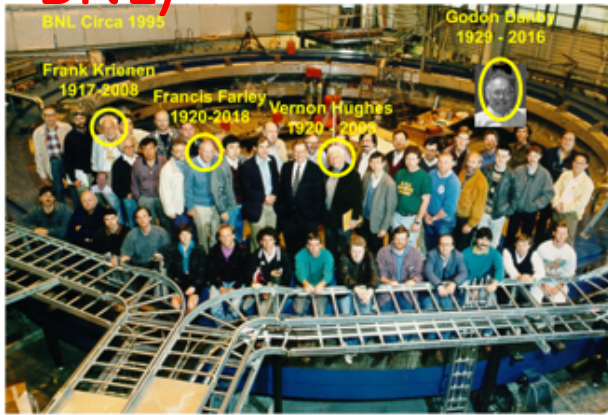
J. Bailey, K. Borer, F. Combley, H. Drumm, C. Eck, F.J.M. Farley, J.H. Field, W. Flegel, P.M. Hattersley, F. Krienen, F. Lange, G. Lebee, E. McMillan, G. Petrucci, E. Picasso, O. Runolfsson, W. von Ruden, R.W. Williams, S. Wojcicki

Key Limitation:

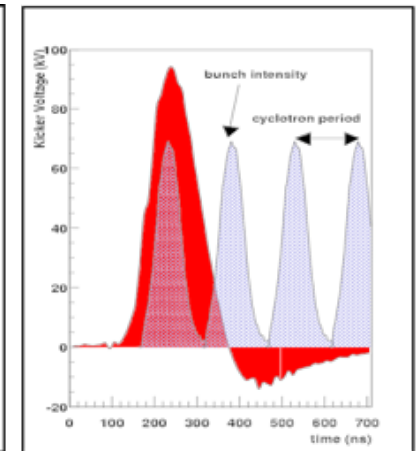
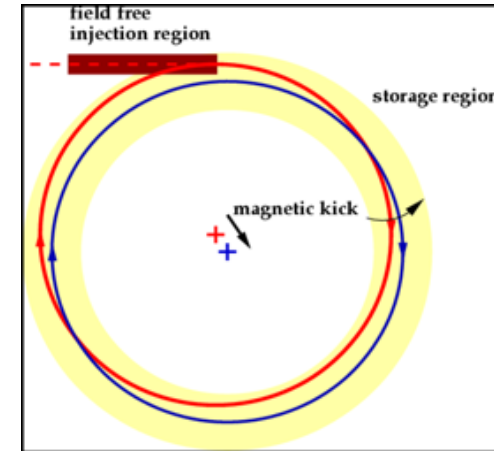
Stored Muons came from injected Pions that “accidentally” decayed in the first turn with kinematics to leave a Muon on orbit

Only 25 ppm chance !!!

With the technique “perfected” (more or less), the next big idea involved getting a higher intensity muon storage: **Direct Muon Injection** (now at **BNL**)

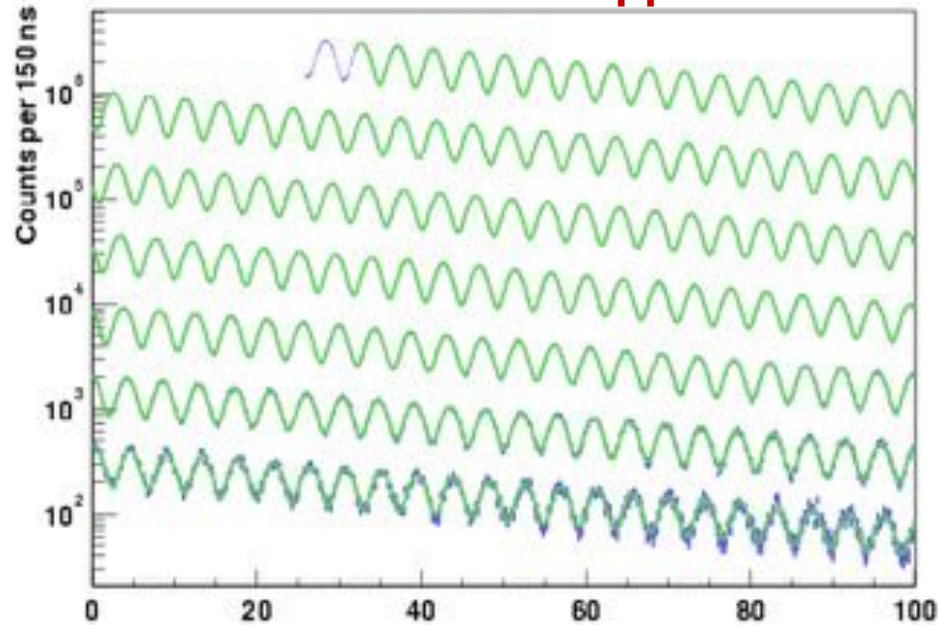


Superconducting Storage Ring

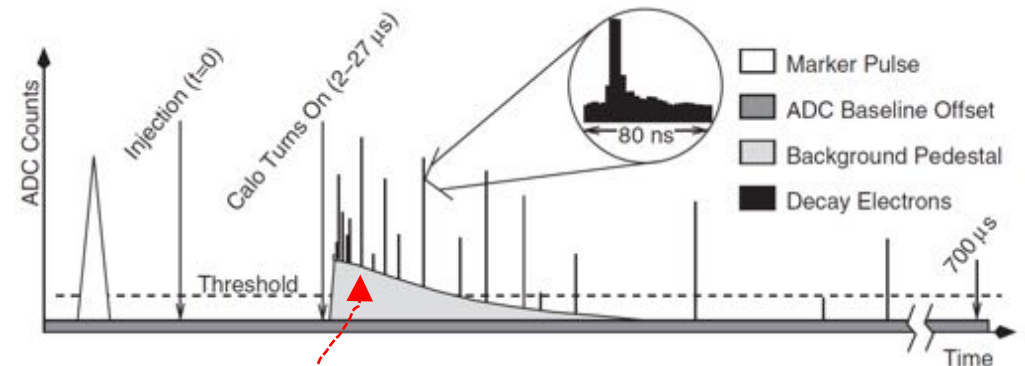


Kick them sideways onto a stored orbit

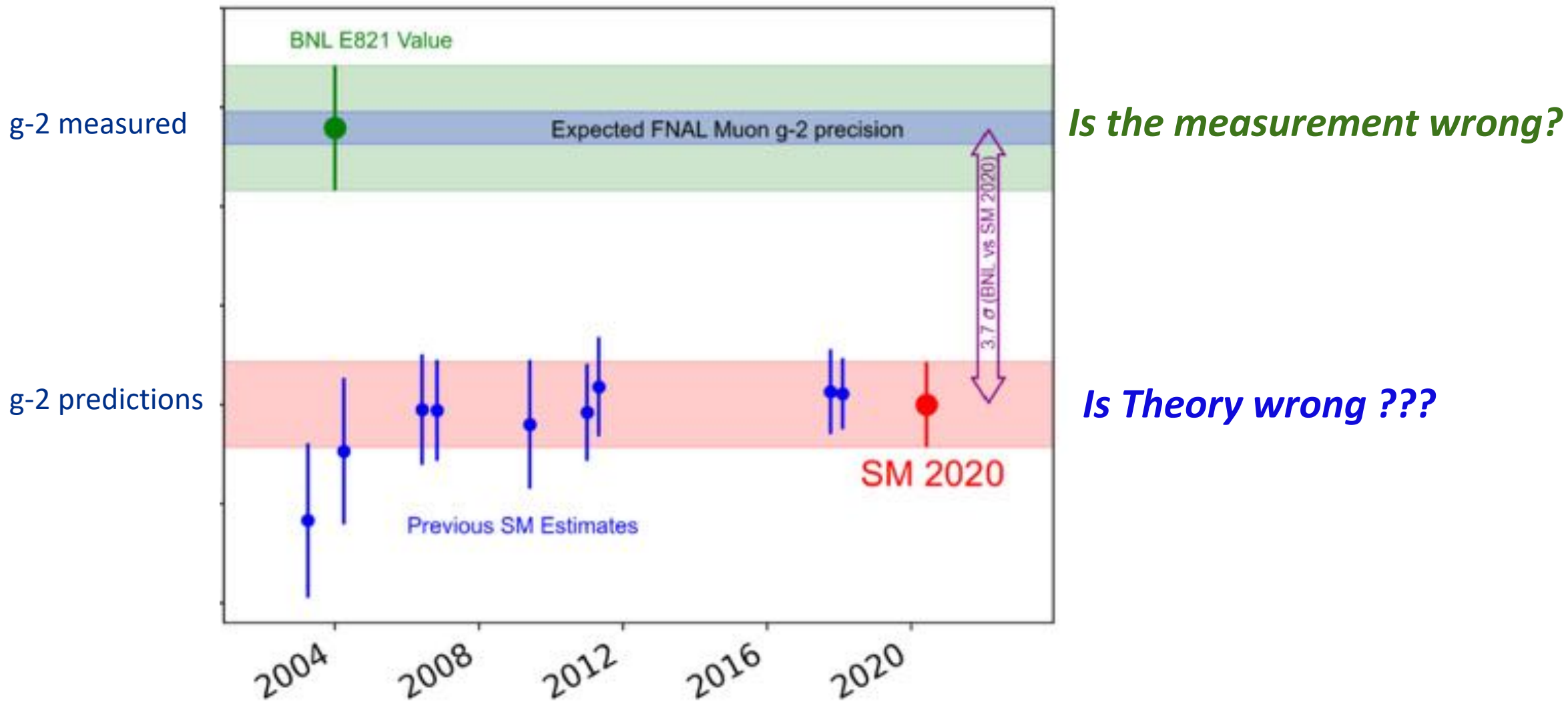
Precision: **540 ppb**



Big problem remained: Short beamline meant pions dominated and caused a huge hadronic flash in the detectors



@540 ppb precision, the BNL measurement remained in tension with Theory, and slightly increasing over 2 decades! **Why?**

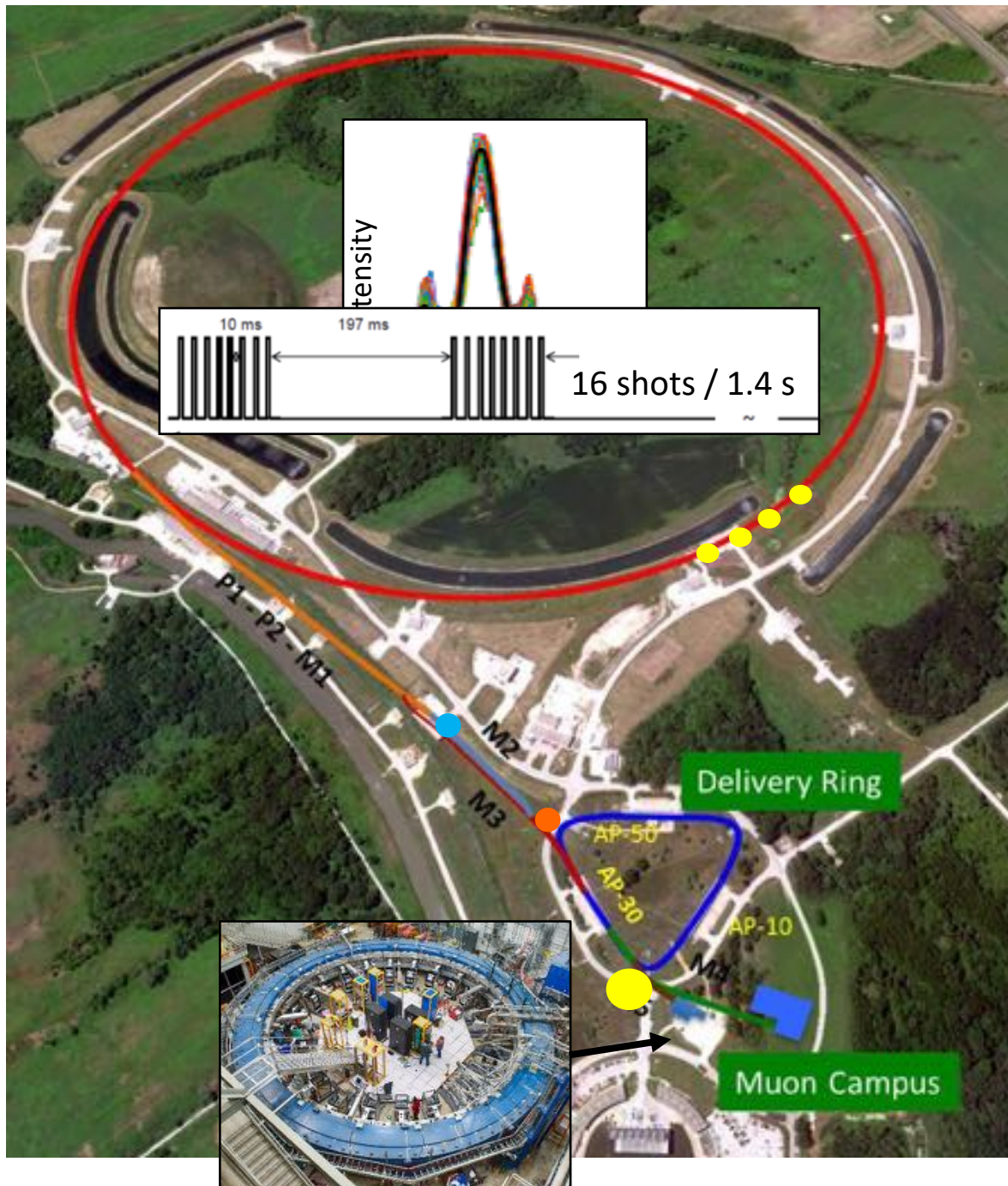


Time for a New and Improved Measurement (to settle the situation)

The Fundamental Experimental Principle is Unchanged, but the FNAL Muon $g-2$ represents significant improvements in all aspects



We include: Particle-, Nuclear-, Atomic-, Optical-, Accelerator-, and Theory Physicists

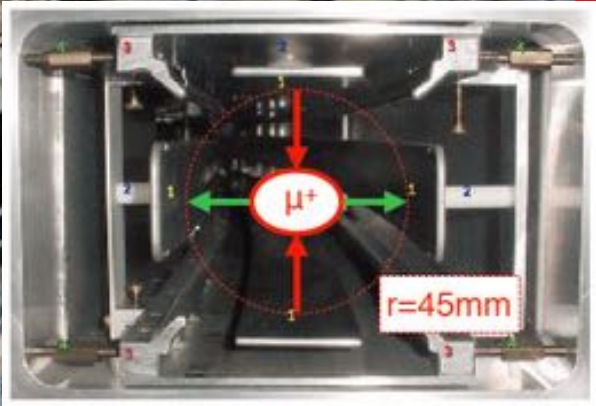


Creating the Polarized Muon Beam for g-2

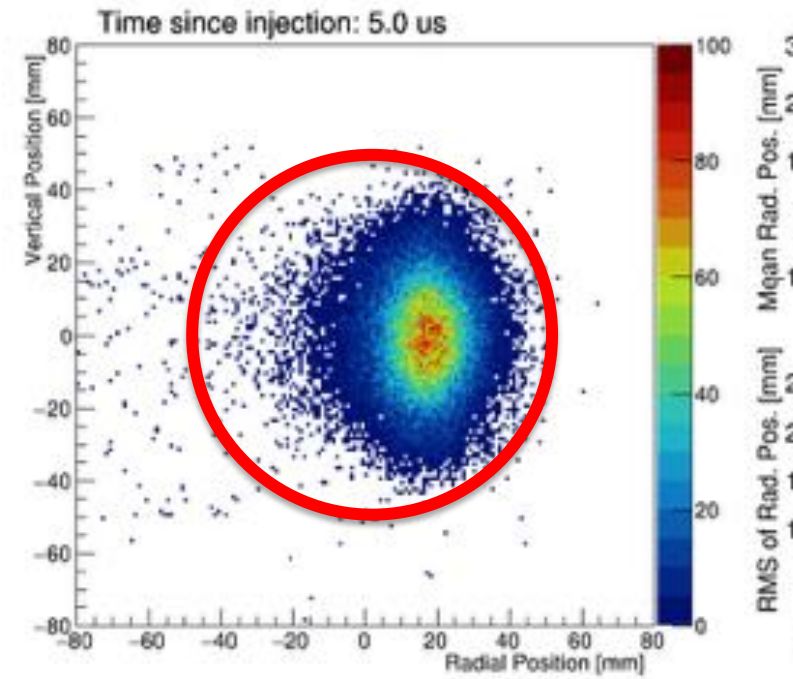
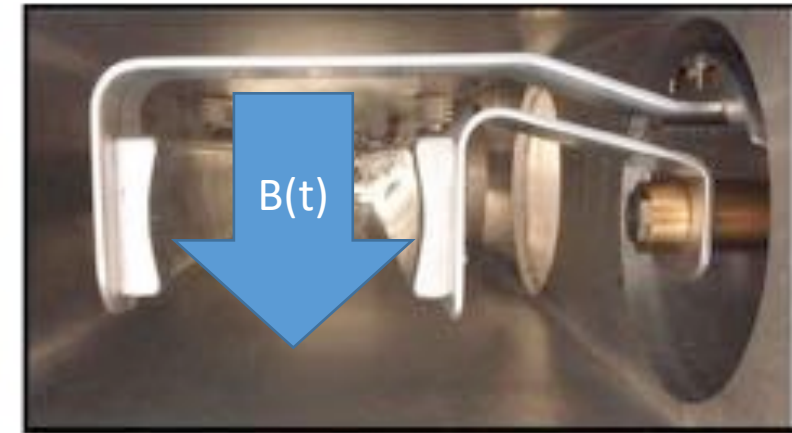
- 8 GeV protons
- Divide in 4 bunches
- Extract each to strike target
- Magnetic lenses collect $\pi \rightarrow \mu\nu$
- $\rho/\pi/\mu$ beam enters Delivery Ring – protons get kicked out; pions decay away
- **And only muons enter storage ring**

Establishing where the muons are stored is imperative. Quadrupoles critical

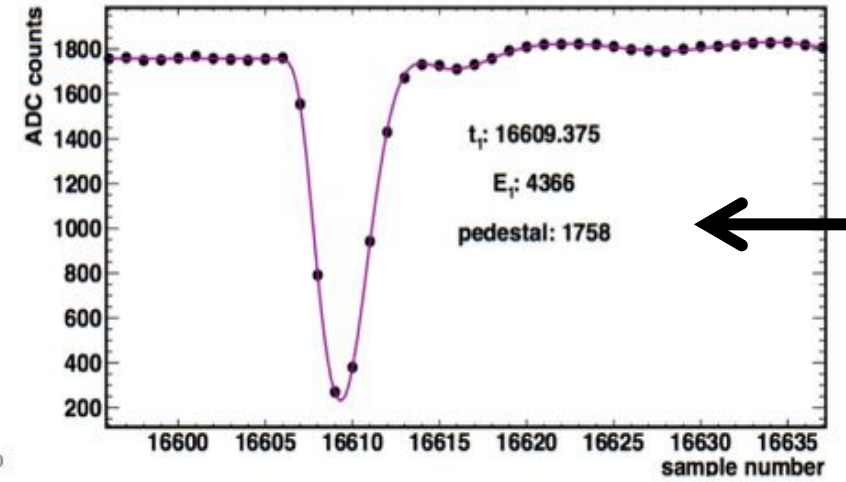
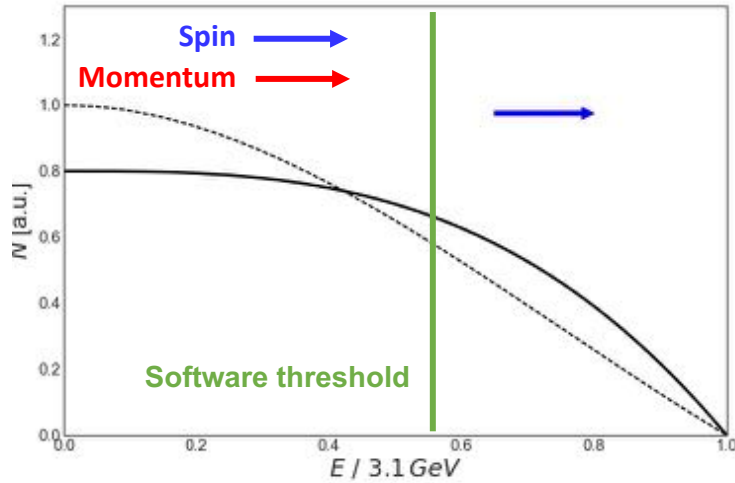
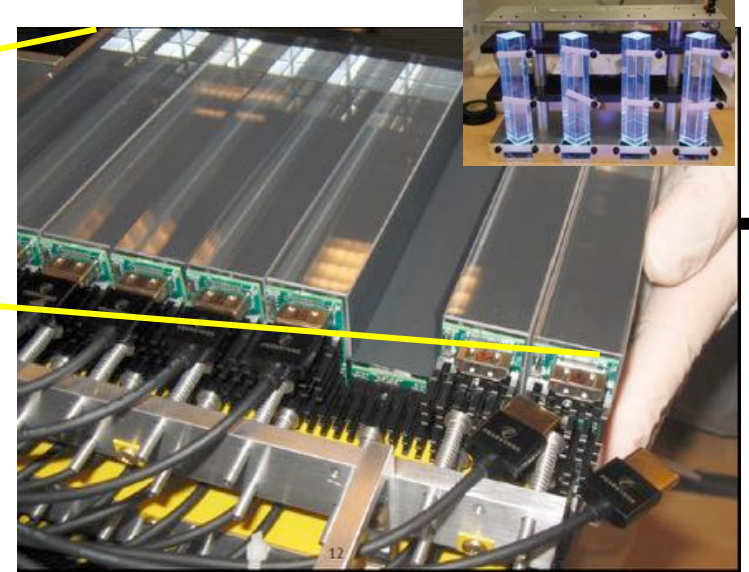
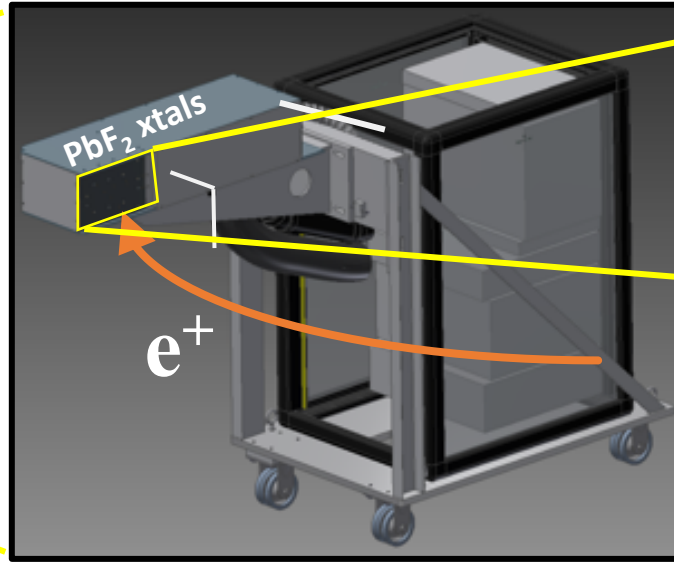
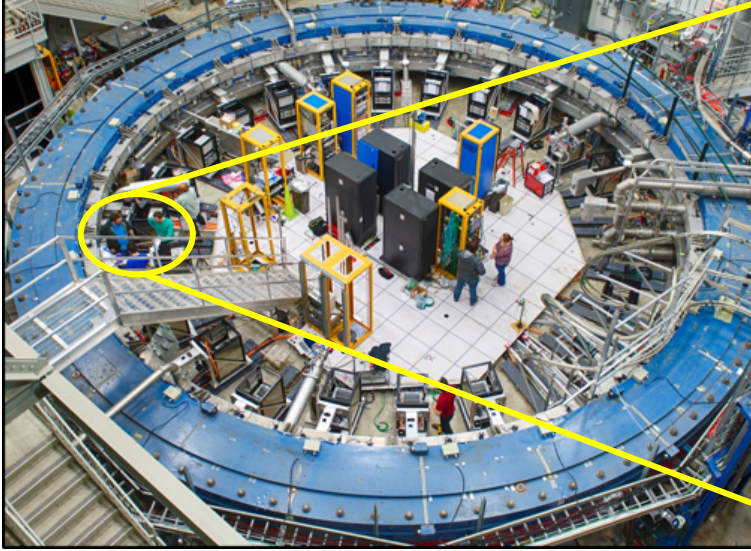
reflector



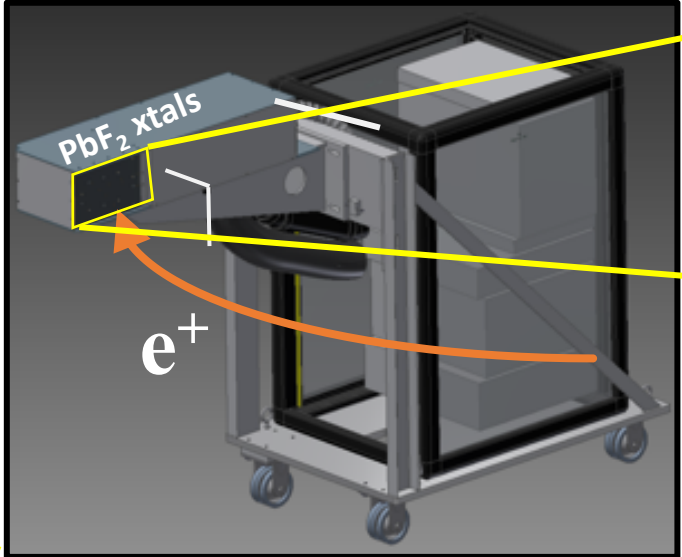
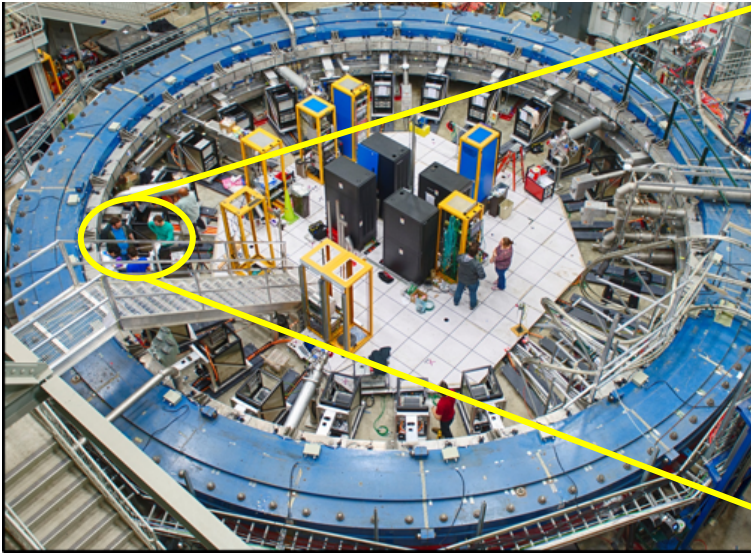
Muons enter and get "Kicked" onto stable orbits



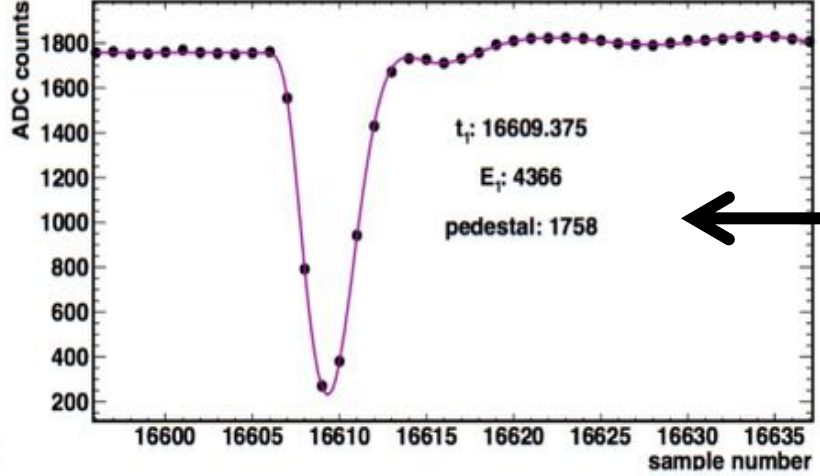
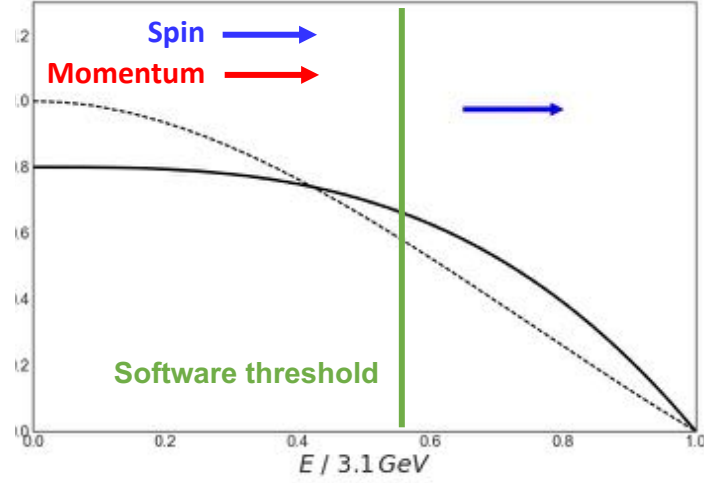
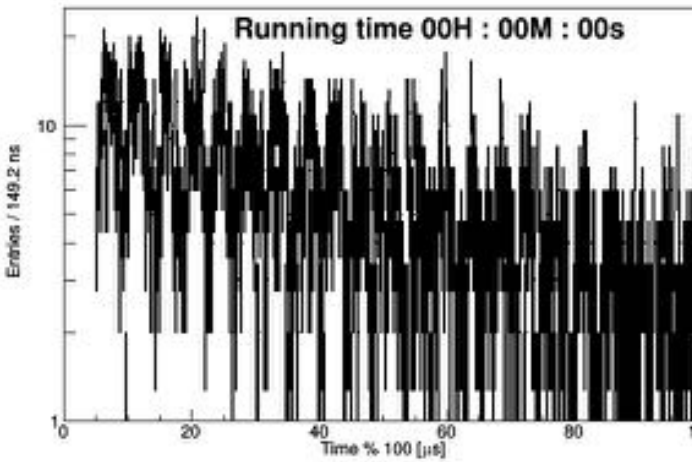
The precession frequency, ω_a is derived from a time histogram of high-energy e^+ decay events



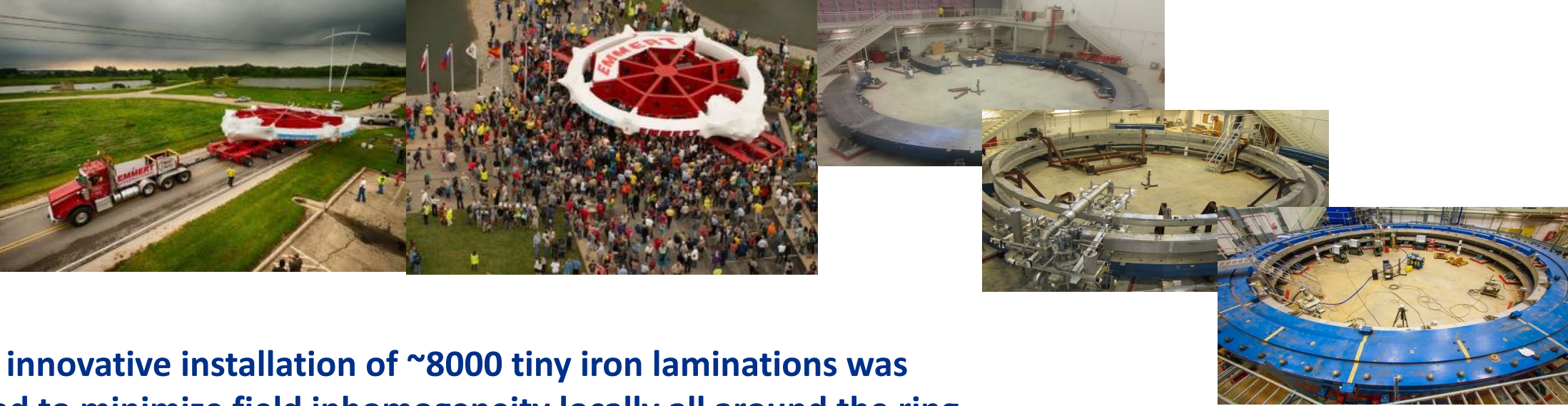
The precession frequency, ω_a is derived from a time histogram of high-energy e^+ decay events



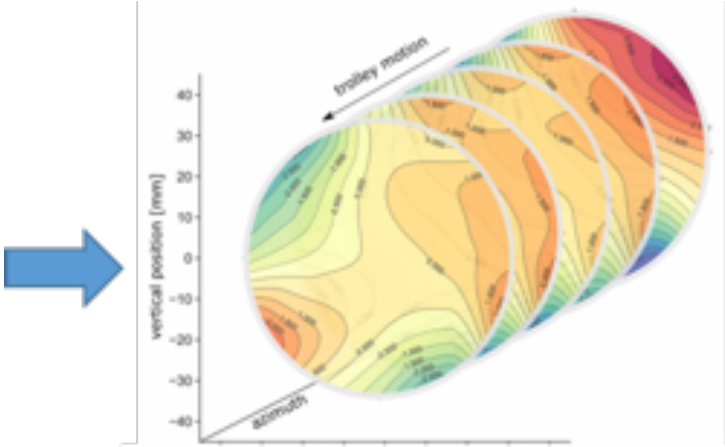
Events above threshold



The Field, ω_p begins with the BNL magnet moved to Fermilab and shimmed and monitored to unprecedented levels



An innovative installation of ~8000 tiny iron laminations was used to minimize field inhomogeneity locally all around the ring

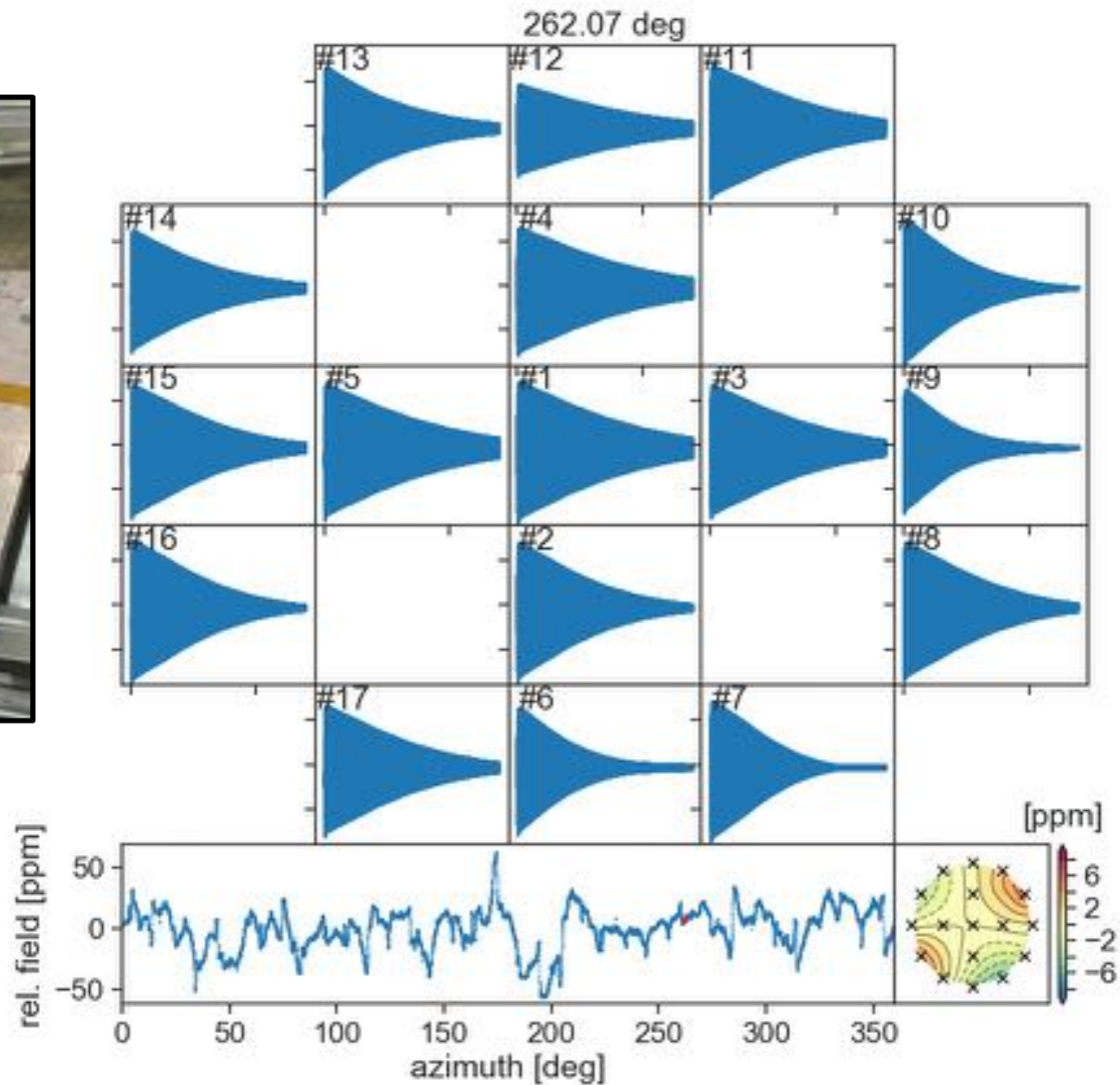
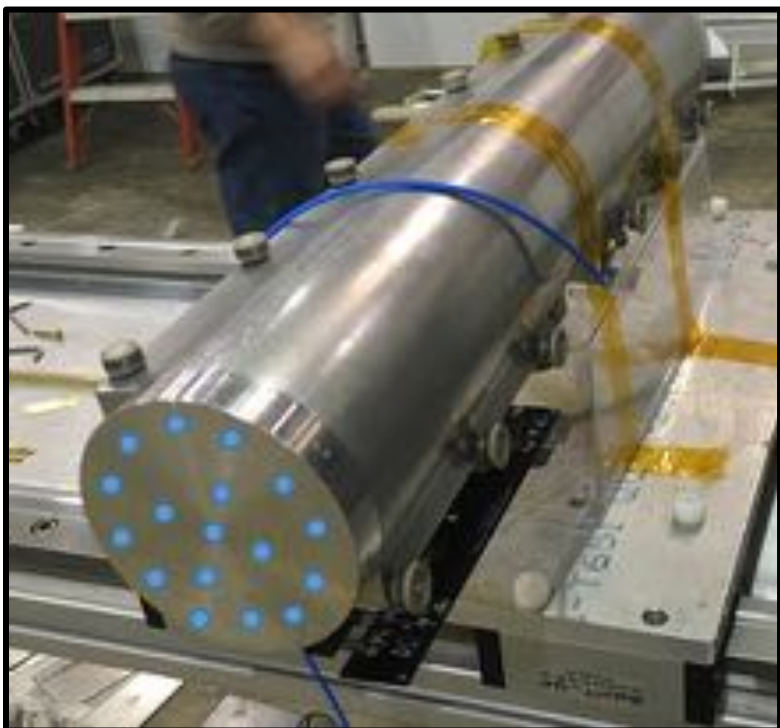


Final field uniformity is ~3 x finer than BNL !

Sequence of field 2D field slices as trolley moves

Taking you on a Trolley Run

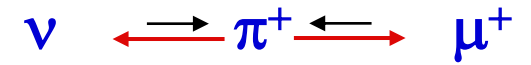
...



5 “miracles permit measurements of g-2 to sub-ppm Precision

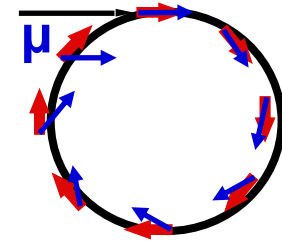
- **Polarized muons produced naturally in pion decay**

~97% for forward decays



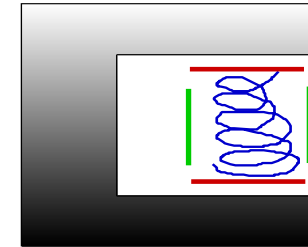
- **Precession frequency is proportional to (g-2)**

Independent of speed (γ) of the muon



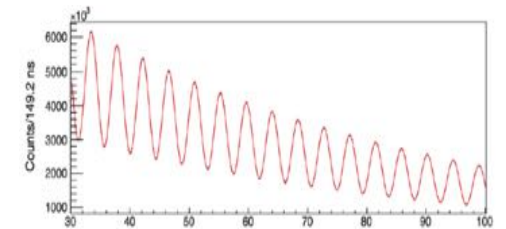
- **P_μ The magic momentum**

The E field does not perturb the spin frequency at 3.094 GeV/c



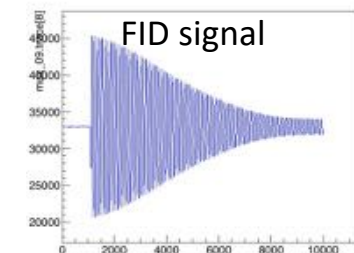
- **Parity violation in the decay $\mu^+ \rightarrow e^+ \nu_e \bar{\nu}_\mu$**

◆ encodes the anomalous precession frequency in e^+ vs time




- **Proton NMR magnetometers**

◆ Continuous determination of the magnetic field throughout the volume in which the muons are stored



a_μ is obtained from the **2 frequency measurements we make**
 ... and well-known fundamental factors from others

 We measure these 2 frequencies

$$a_\mu = \left[\frac{\omega_a}{\tilde{\omega}'_p(T_r)} \right] \underbrace{\left[\frac{\mu'_p(T_r)}{\mu_e(H)} \frac{\mu_e(H)}{\mu_e} \frac{m_\mu}{m_e} \frac{g_e}{2} \right]}$$

$\frac{\mu_e(H)}{\mu'_p(T)}$ Measured to 10.5 ppb at $T = 34.7^\circ\text{C}$
 Metrologia 13, 179 (1977)

$\frac{\mu_e}{\mu_e(H)}$ Bound-state QED (exact)
 Rev. Mod. Phys. 88 035009 (2016)

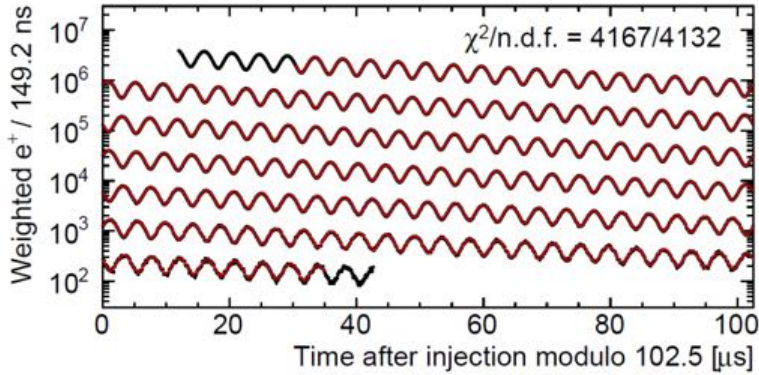
$\frac{m_\mu}{m_e}$ Known to 22 ppb from muonium hyperfine splitting
 Phys. Rev. Lett. 82, 711 (1999)

$\frac{g_e}{2}$ Measured to 0.28 ppt
 Phys. Rev. A 83, 052122 (2011)

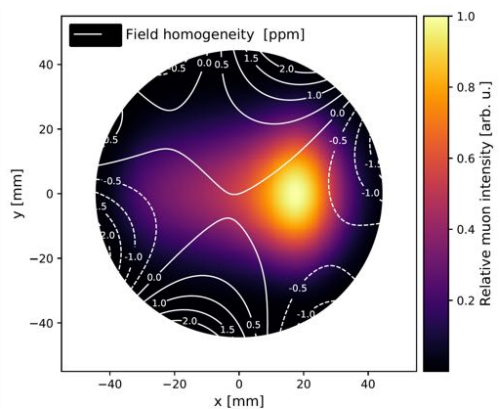
Many measurements determine a_μ . This is our working analysis recipe (but I'll spare you the details behind each term)

$$a_\mu \propto \frac{f_{\text{clock}} \omega_a^m (1 + C_e + C_p + C_{ml} + C_{pa})}{f_{\text{calib}} \langle \omega'_p(x, y, \phi) \times M(x, y, \phi) \rangle (1 + B_k + B_q)}$$

- f_{clock} • Blinded clock
- ω_a^m • Measured precession frequency
- C_e • Electric field correction
- C_p • Pitch correction
- C_{ml} • Muon loss correction
- C_{pa} • Phase-acceptance correction



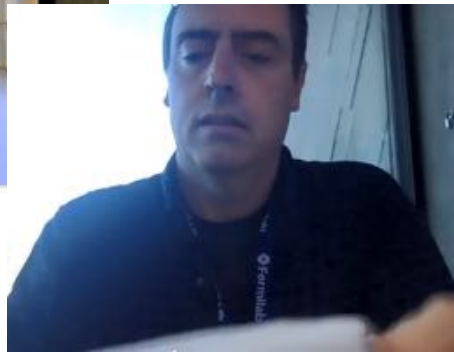
-
- f_{calib} • Absolute magnetic field calibration
 - $\omega'_p(x, y, \phi)$ • Field tracking multipole distribution
 - $M(x, y, \phi)$ • Muon weighted multipole distributed
 - B_k • Transient field from the eddy current in kicker
 - B_q • Transient field from the quad charging



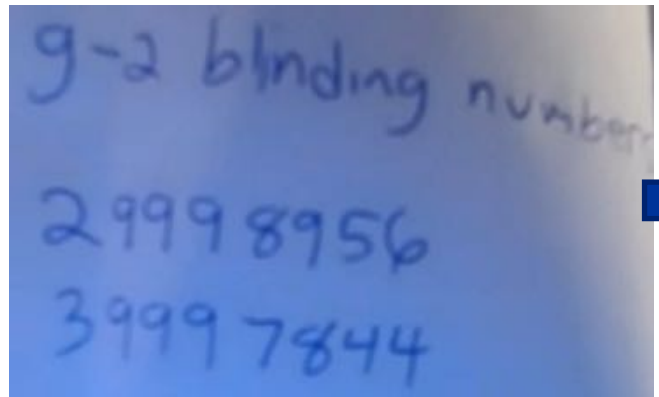
The First "Unblinding" in 2021 (6% of the full data set)



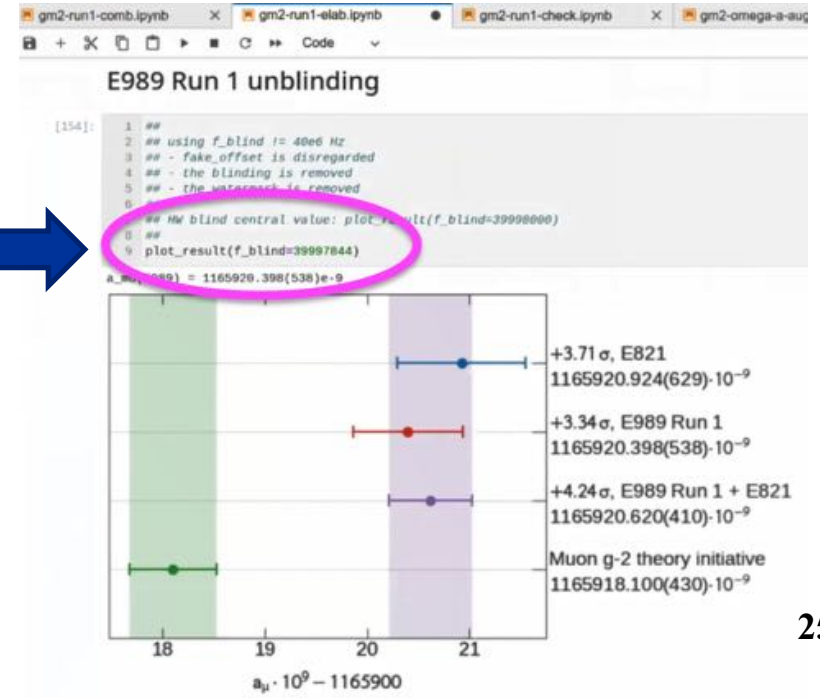
UW envelope



FNAL envelope



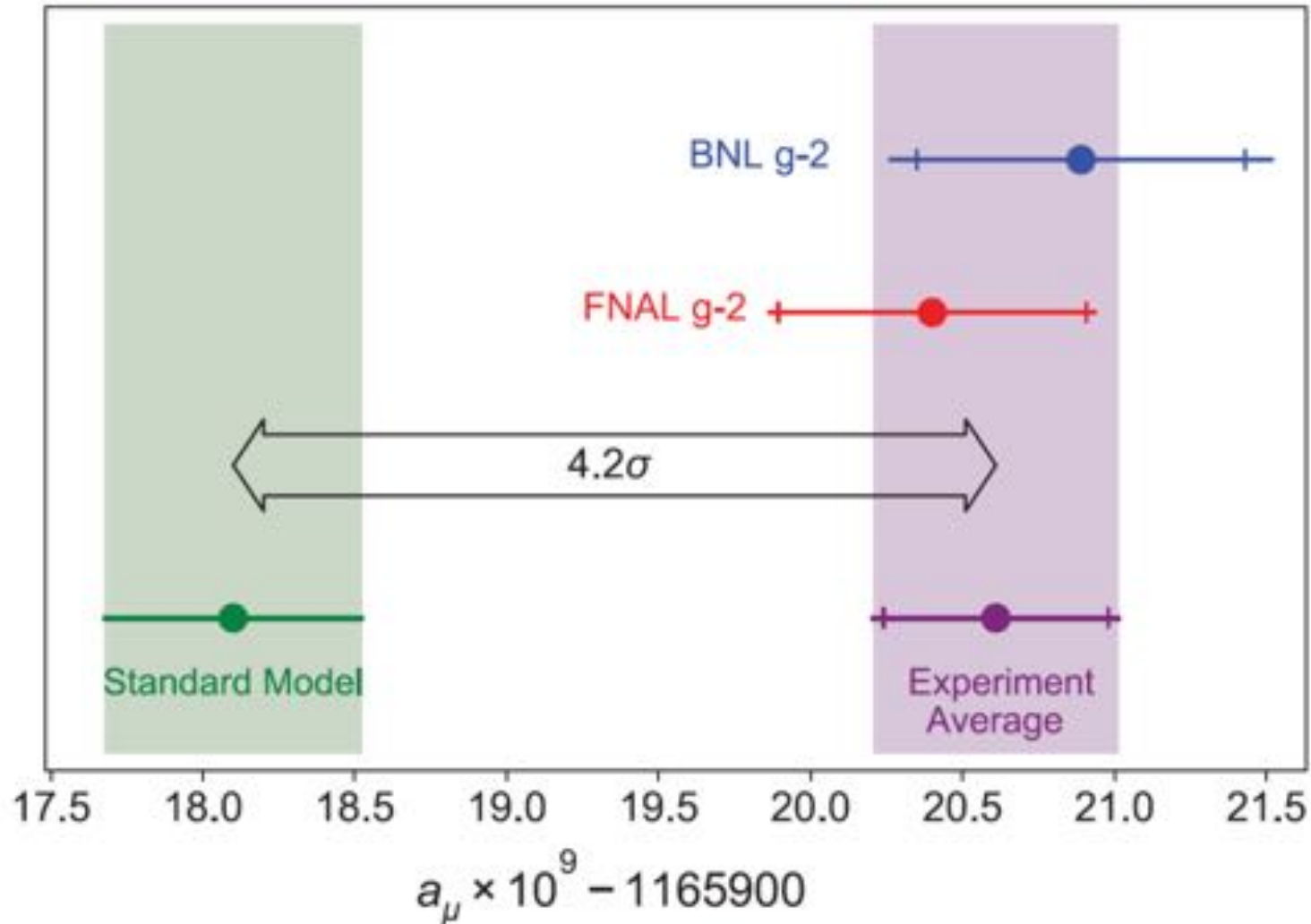
Same numbers!



It means:

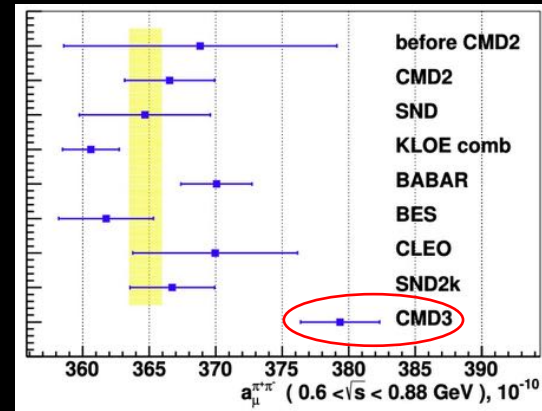
1) The 20-year-old BNL result is confirmed

2) The combined discrepancy with the Standard Model increases to 4.2σ



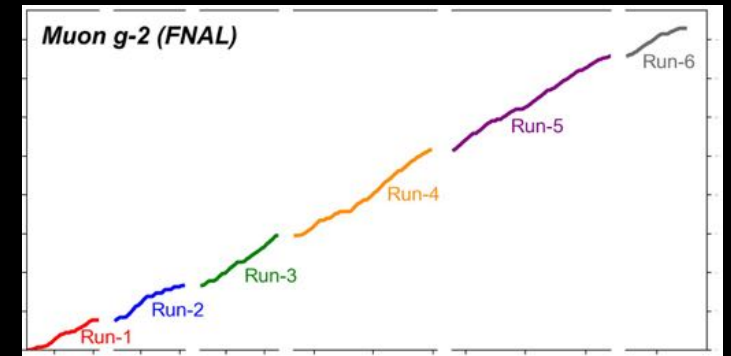
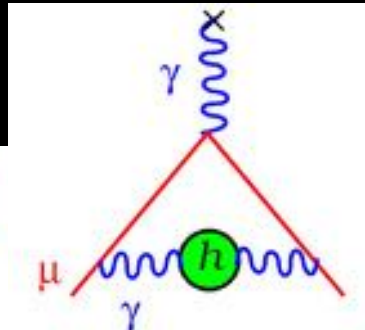
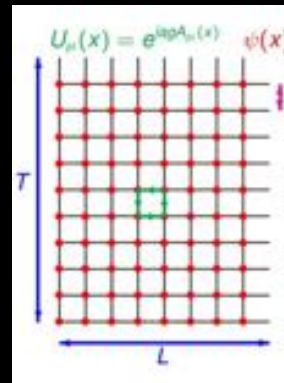
The Aftermath, the Near, and Far Future

Supersymmetry
 Lepto-quarks
 BSM?
 Axion-like particles
 Two-Higgs Doublet Model



New $\pi\pi$ data

Lattice HVP



Next $g-2$ Results

The 2021 Result generated many creative BSM explanations

Supersymmetry

Lepto-quarks

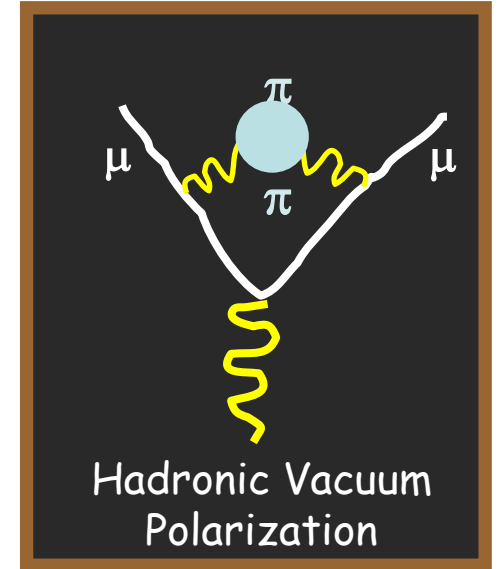
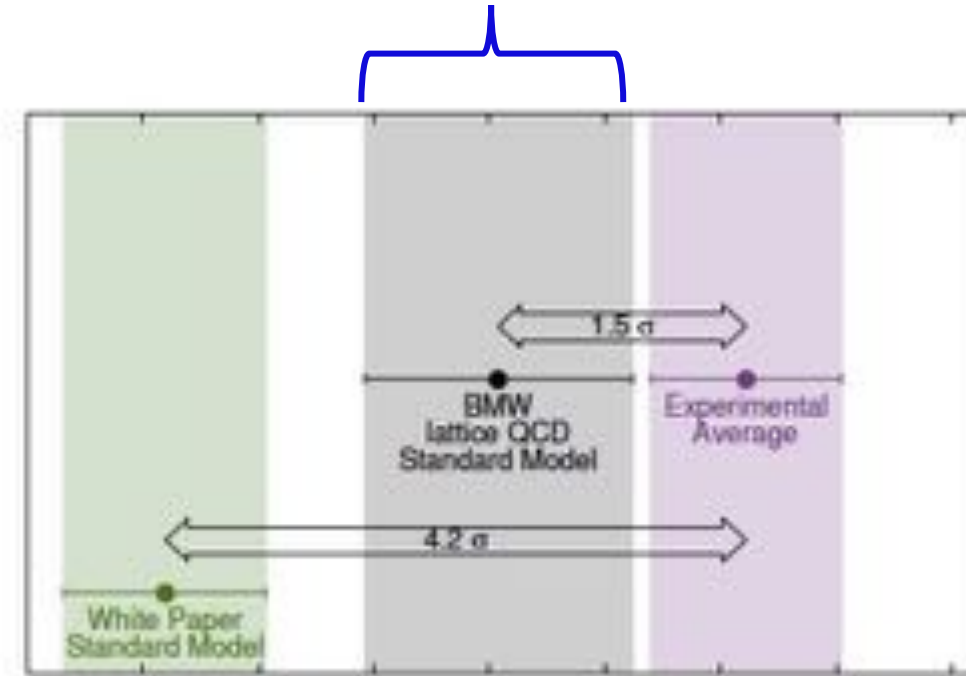
Axion-like
particles

Two-Higgs
Doublet Model

Most agree that it's not easy to explain the results without
"tuning" their models more than one might wish

But, others believe the Standard Model might be *wrong* ☹️

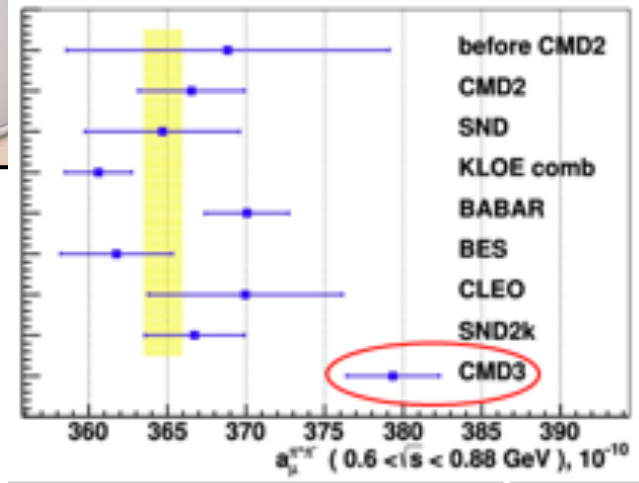
A Lattice QCD team calculated HVP and got a different result, shown in gray here



g-2

At present, many other LQCD groups are working on the same calculation, with intermediate step comparisons and blinded techniques. Stay tuned

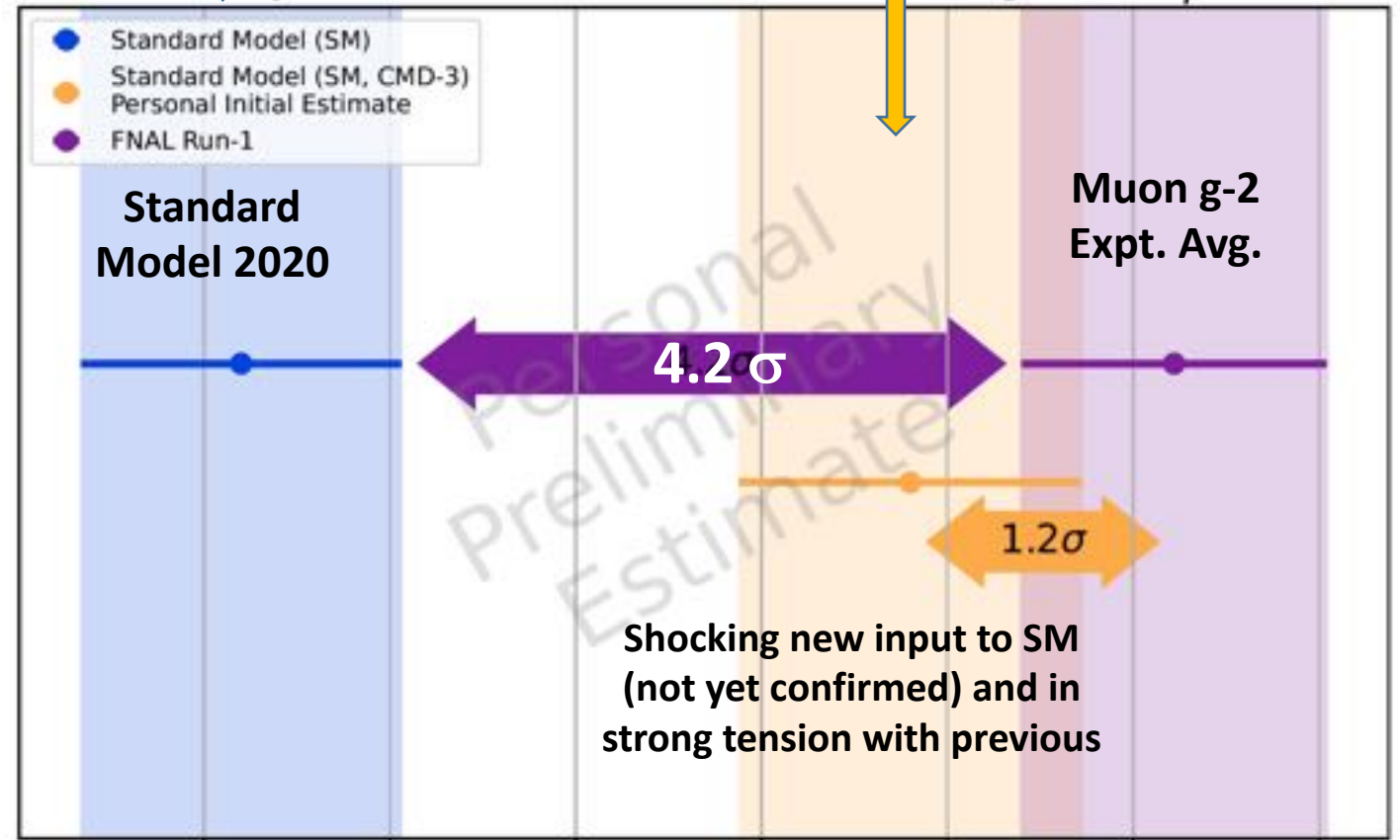
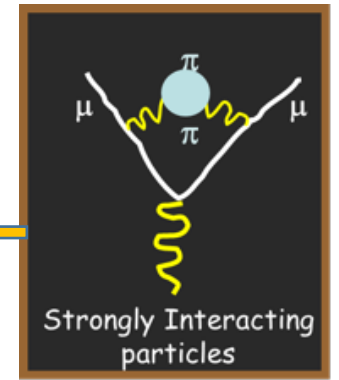
On the data side, CMD-3 surprised the world



New $\pi\pi$ data

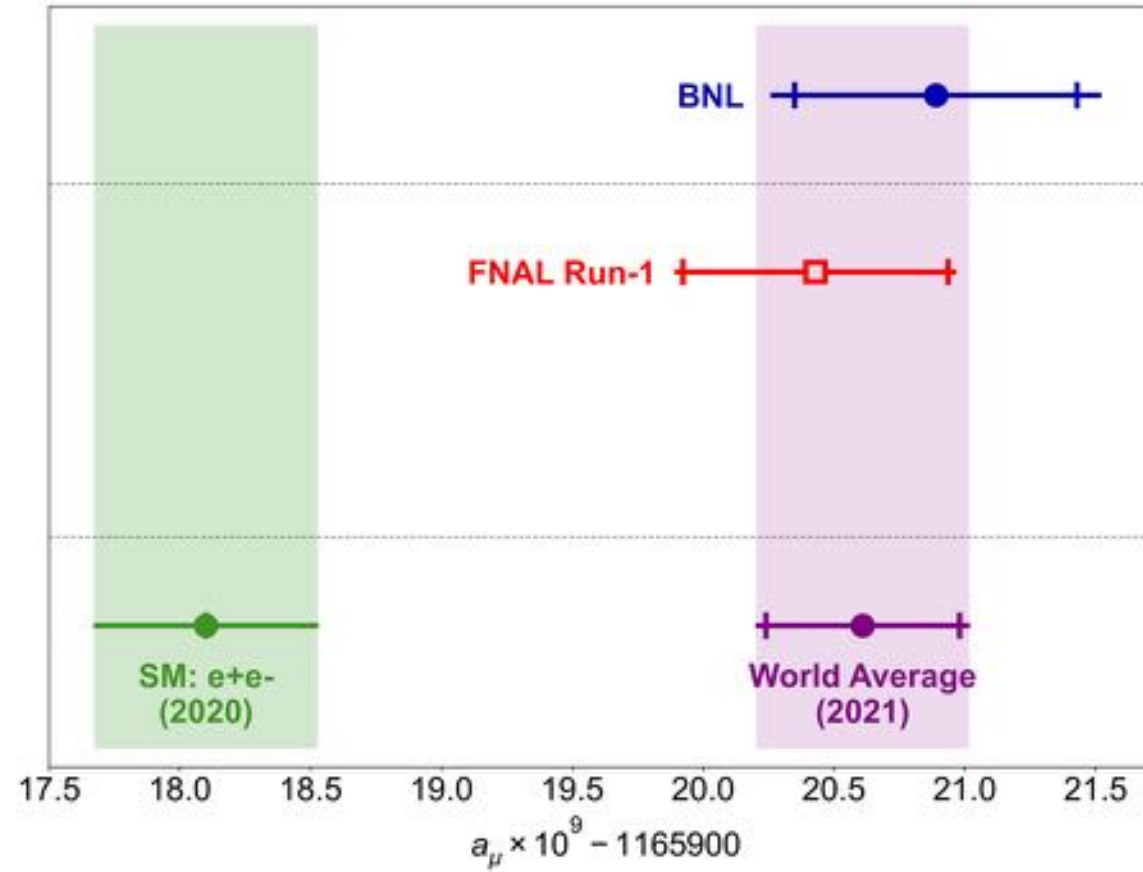
Remember: This Diagram is evaluated using **DATA** other experiments

The community is bewildered !
Their new data **disagrees with all previous experiments**, including their own!

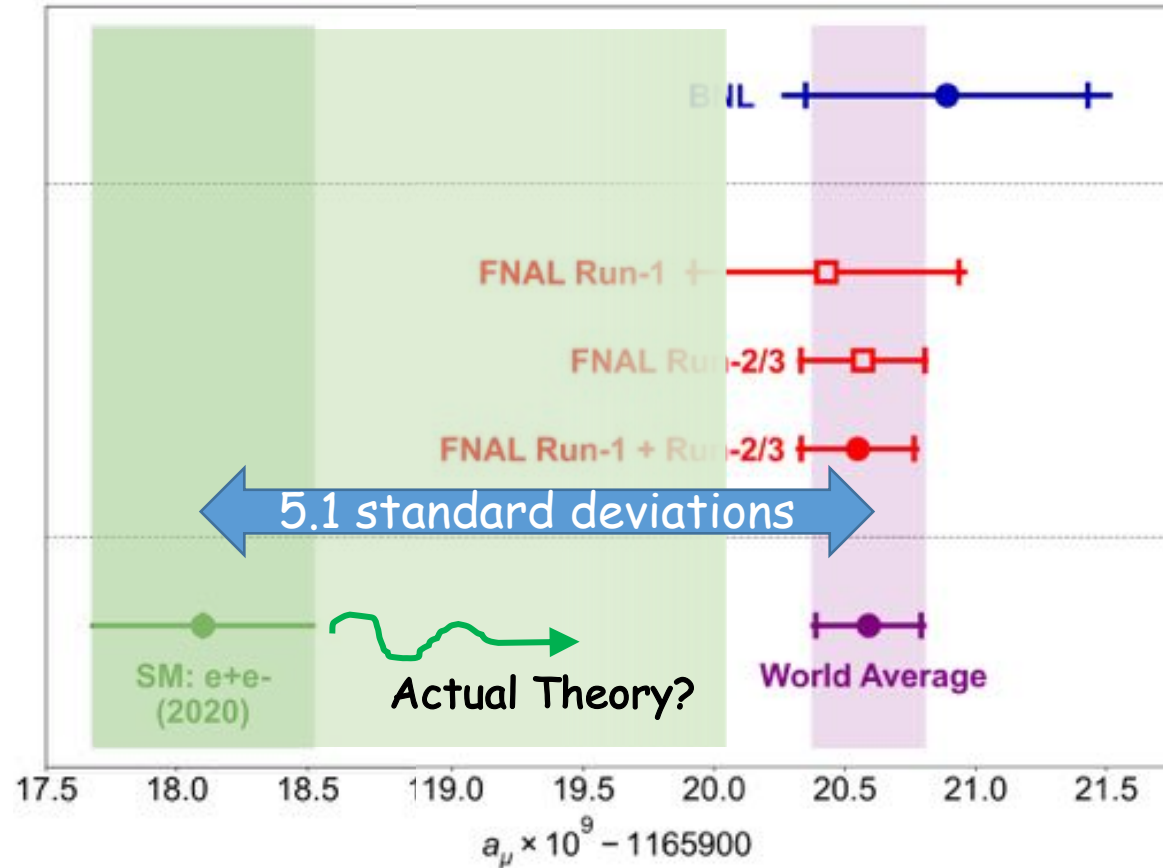


g-2

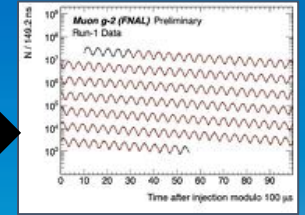
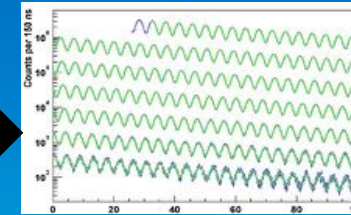
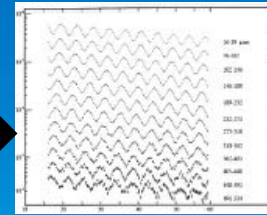
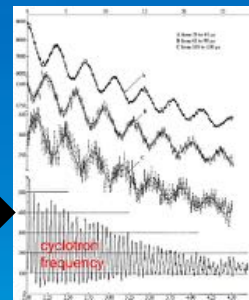
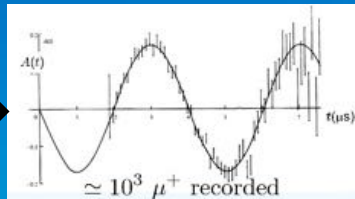
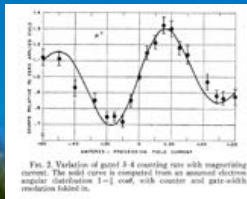
Our 2023 result: with half the uncertainty and bettering our systematic proposal budget already



Our 2023 result: with half the uncertainty and bettering our systematic proposal budget already



The Chess Game continues !!



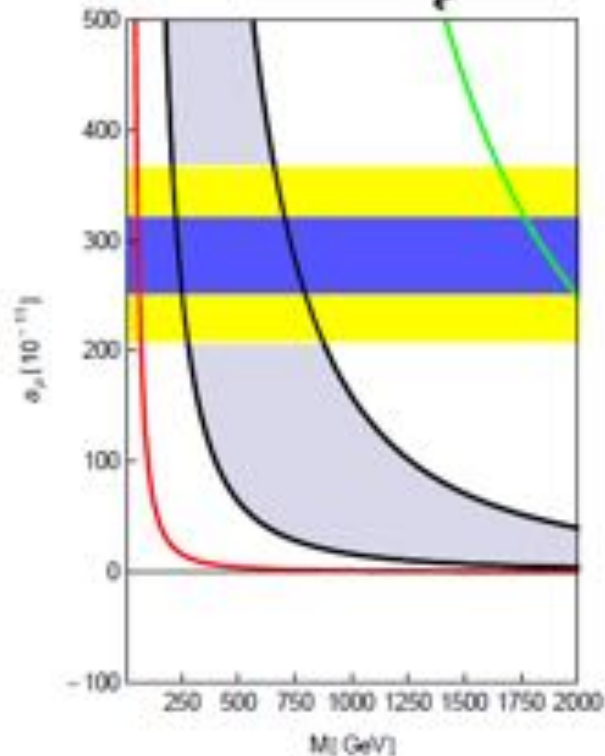
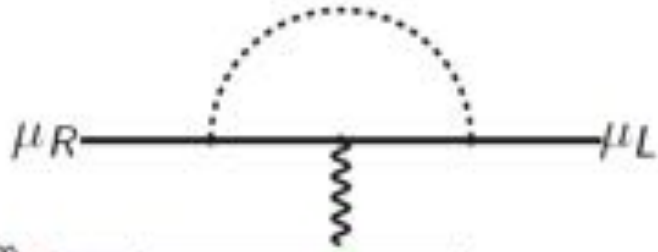
50+ Years of Muon “g-2” experiments

- I believe Stanley would enjoy this ebb and flow of precision experimental work and high-level theory
- The road is long, can be hard and bumpy, but ultimately we are seeking Nature’s truth
- On our side, we have one more big data set to release ... 2025 is the target
- Until then, there is a vigorous SM Theory campaign ongoing to determine the hadronic contributions

In a generic sense, these are “loop effects” that couple to the muon mass and moment in similar fashion, characterized by C , a coupling:

$$\mathcal{O}(C) \left(\frac{m_\mu}{M}\right)^2$$

$$C = \frac{\delta m_\mu(\text{N.P.})}{m_\mu}$$



$\mathcal{O}(1)$

radiative muon mass generation ...

[Czarnecki, Marciano '01]

[Crivellin, Girrbach, Nierste '11][Dobrescu, Fox '10]

$\mathcal{O}\left(\frac{\alpha}{4\pi} \dots\right)$

supersymmetry ($\tan \beta$)

vectorlike fermions ...

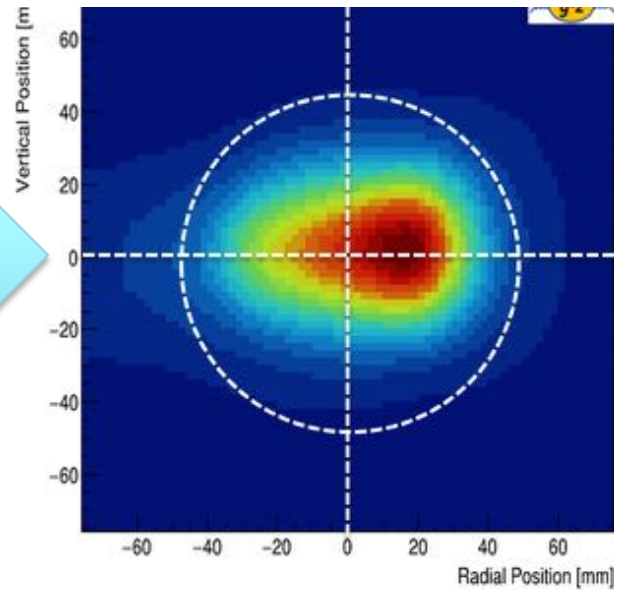
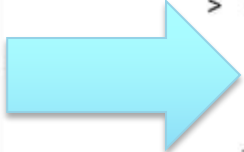
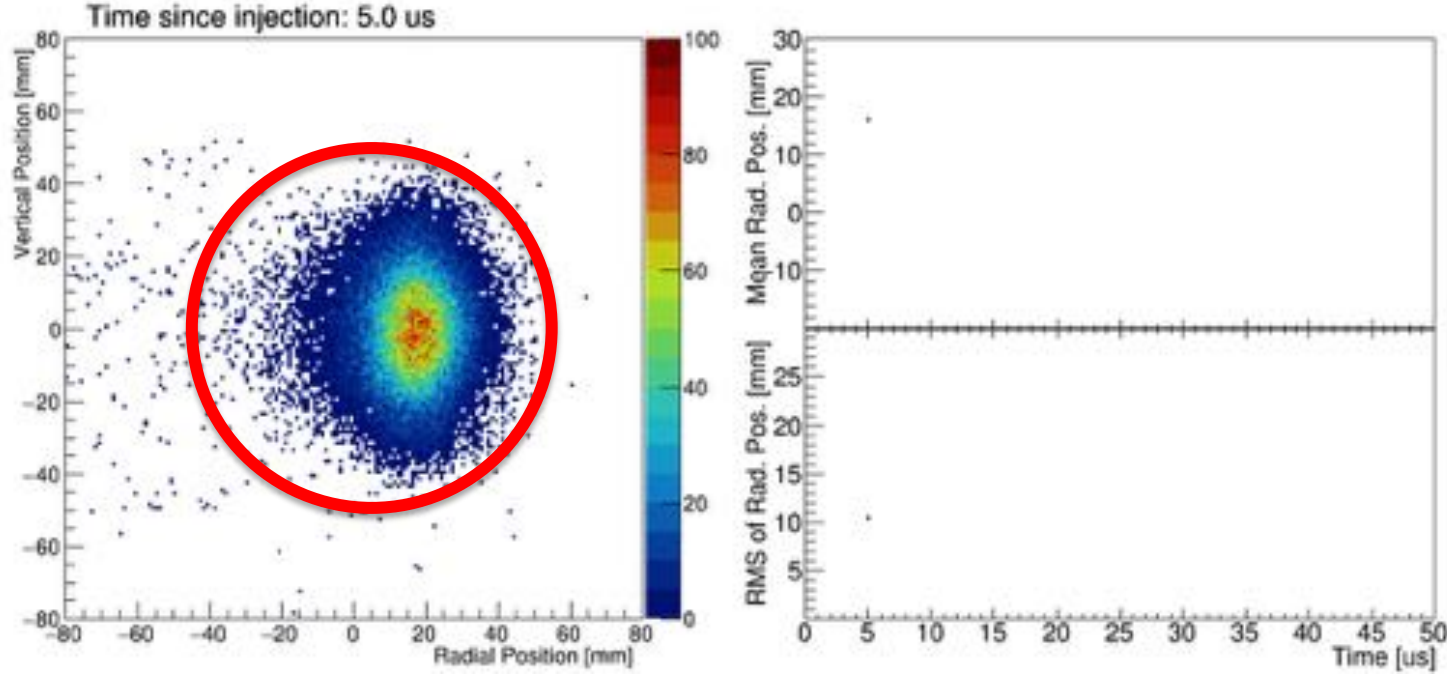
$\mathcal{O}\left(\frac{\alpha}{4\pi}\right)$

SM: Z, W . New physics: $Z', W' \dots$

$< \frac{\alpha}{4\pi}$

2-Higgs doublet model, dark photon .

An example looking “inside” the storage ring at the dynamic motion of the muons as they go around... which adds to the complexity



Average x-y profile around the ring

